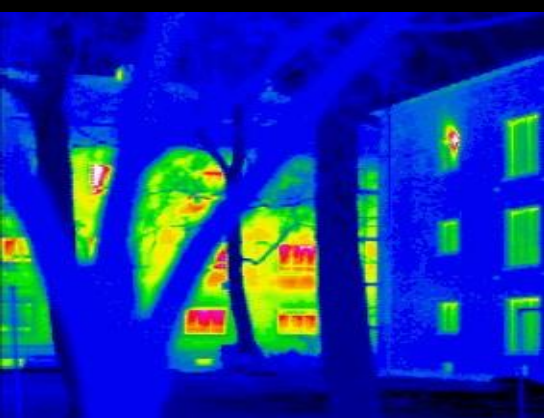
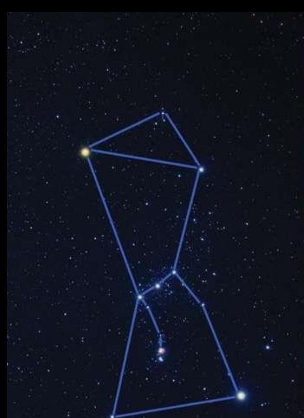
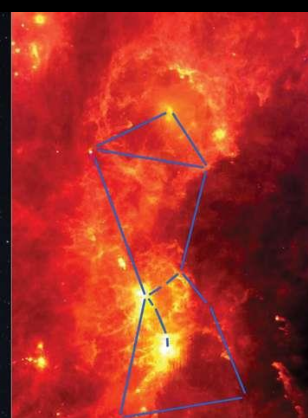
National Aeronautics and Space Administration

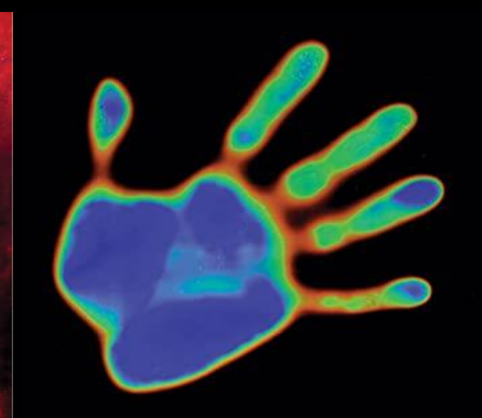
High School Experiments in Infrared Astronomy

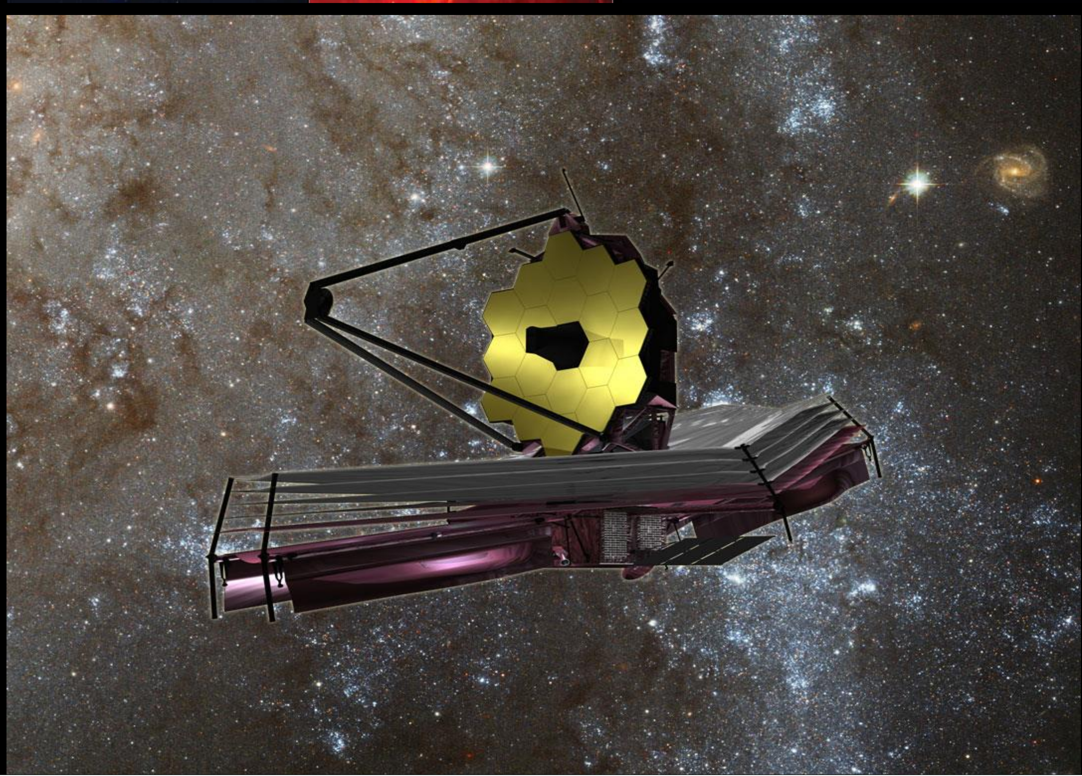
Featuring the James Webb Space Telescope











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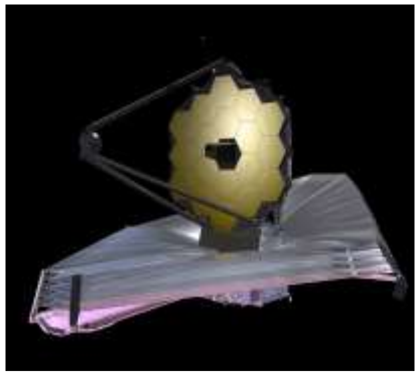

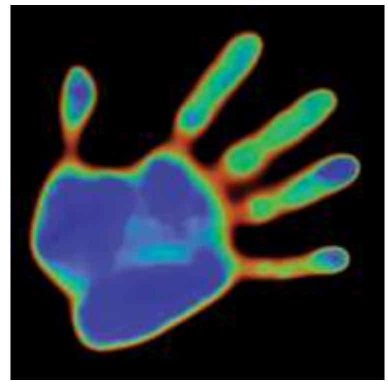

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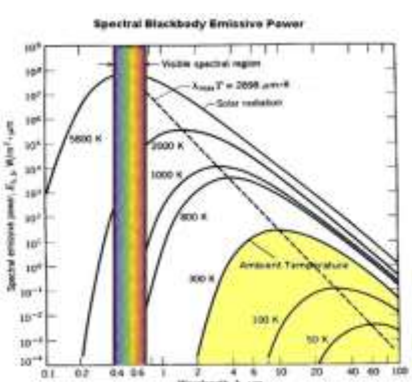

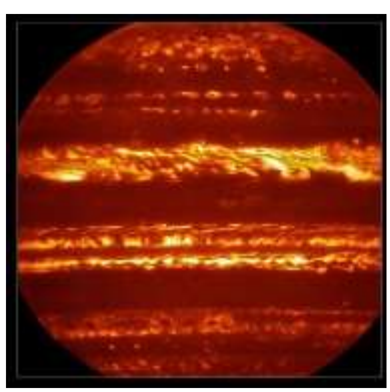

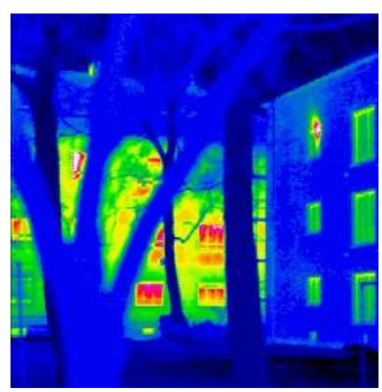

High School Experiments in Infrared Astronomy

An Introduction to the science of the Webb Space Telescope including hands-on experiments and worked math exercises

Dr. Sten Odenwald (NASA/HEAT)

Developed by SpaceMath@NASA for the

Heliophysics Education Activation Team

Goddard Space Flight Center

Greenbelt, Maryland

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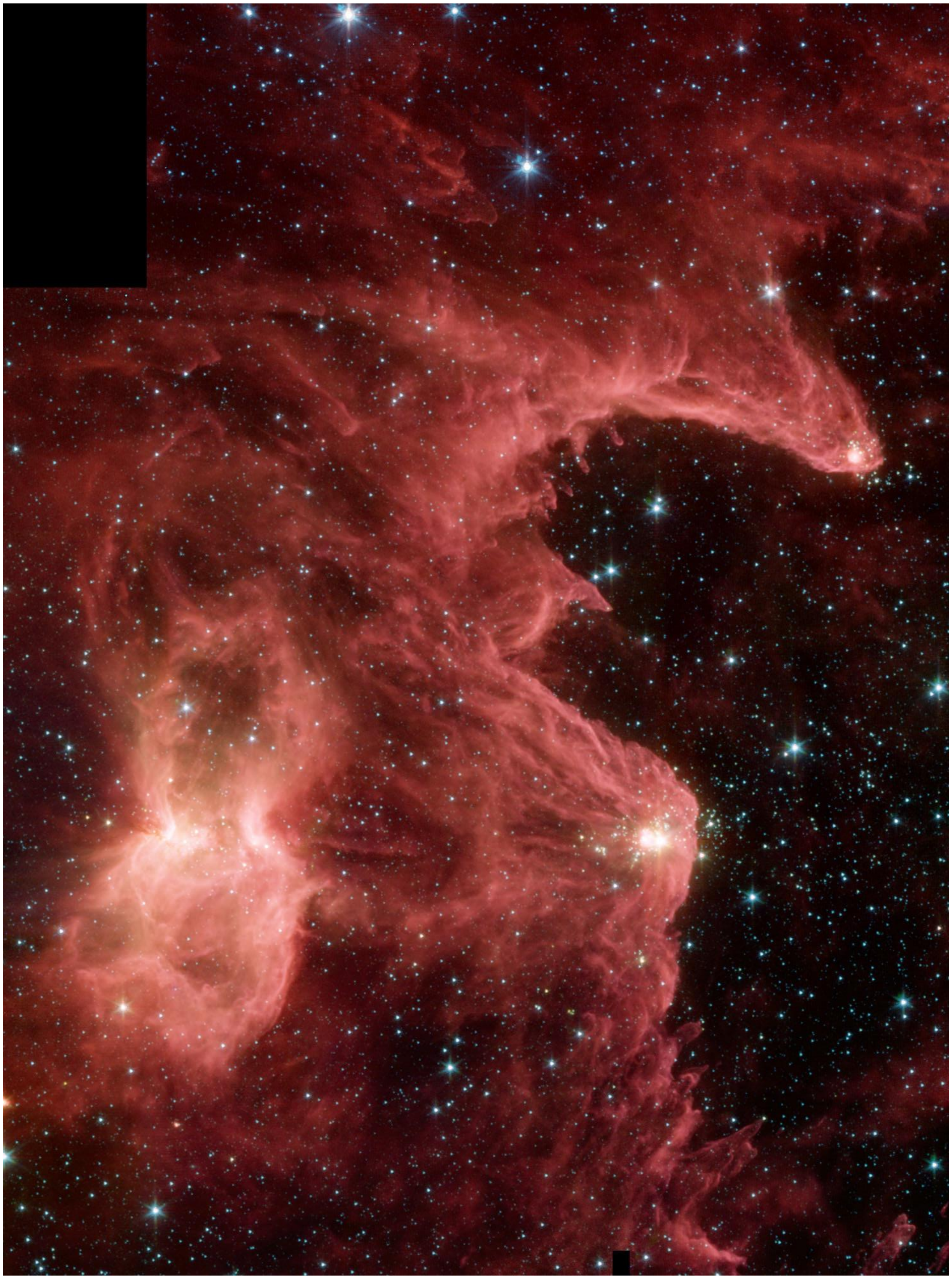

Infrared image of the W5 star-forming region in Perseus (Credit: NASA/Spitzer)

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Introduction to this Guide

The James Webb Space Telescope (JWST) will be launched in late-2021. It will journey through space to its destination at the L2 orbital position located 1.5 million kilometers from Earth. There, it will unfold its segmented mirror and after six months of checkouts, begin its historic investigations of the universe. Unlike optical telescopes on Earth, and the Hubble Space Telescope in orbit, JWST will only operate at infrared wavelengths where the 'heat' from distant objects can be used to form images of them. The Mid-Infra Red Imager (MIRI), for example, is sensitive enough to detect a candle on one of the moons of Jupiter!

Infrared imaging is a technique familiar to 'night vision' technology that can locate warm objects at night-time through the heat that they emit. Infrared astronomy has been utilized since the 1960's to similarly image distant planets, stars and galaxies to investigate their properties that are not discernable at optical wavelengths. Among the most exciting aspects of this technology is that infrared light can penetrate interstellar dust clouds revealing the young stars hidden within them. At far infrared wavelengths, these astronomical images can even detect the heat from the interstellar clouds themselves, such as the image of the W5 star-forming region at the beginning of this Guide.

This Guide will present the educator, student and life-long learner with a variety of basic information about infrared light, including hands-on experiments and math problems that will quickly make you an 'Expert' in the basic science being carried out by JWST. It will also present information gathered by the predecessors to JWST such as the Spitzer Space Telescope. Many of the Spitzer images will be similar to what JWST will produce, but with JWST's larger mirror, objects revealed by JWST will be discernable from far-greater distances across the cosmos.

An Overview of Relevant Education Standards

Educators will find this guide a helpful adjunct to the concepts being taught at high school-level in the areas of light, space science and applied mathematics. Table 1 indicates the specific NGSS standards covered by the individual experiments while Table 2 indicates the math skills and science topics covered by the problems. Math problems are listed by highlighted title in the body of the Guide. Chapter VII provides the complete list of problems, which may be printed and distributed to students. Chapter VIII provides the answers for the problems, including details on how to solve them.

Table 1 Connection to NGSS by Experiment

Experiment	**Page**	**Standards**
A- Exploring black body curves	56	PS4.A, PS4.B
B- Your smartphone as an astronomical imager	57	ESS1.A
C- Exploring telemetry math with your smartphone	60	PS4-2
D- Smartphone thermal imaging in far-red band	61	PS4-4
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F- Exploring non-contact infrared thermometers	65	PS4-5
G- A simple IR transmitter and receiver	66	PS4-5, PS4.C
H- Exploring Wien's Displacement Law	68	PS4.B
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J- Infrared mapping	71	PS4.C
K- Exploring electromagnetic radiation	74	ESS1-3
L- Spectroscopy with your car radio!	77	ESS1-3

Table 2 – Math problems and skills

Problem	Skills	Topic
1	C = W x F	Wavelength & frequency
2	Scale models	Engineering
3	Temp conversion	K and C to F
4	C=T/L	Wein Law
5	Calculus	Planck function
6	Working with $L=4\pi R^2\sigma T^4$	Star power
7	Density, mass, volume	Dust emission
8	Calculus	Derive $L=\sigma T^4$
9	Interpreting images	Thermal imaging
10	Interpreting figures	Filter transmission
11	I = P x F	Light transmission
12	Acceleration, scale model	Rocket launch
13	Area of hexagons	JWST mirror
14	Hexagonal geometry	JWST mirror
15	Symmetry	JWST mirror
16	Area and cost	JWST mirror
17	Area and power	Satellite solar power
18	Area, resolution	Digital cameras
19	D= rate x time	Telemetry data rate
20	Unit conversions	Unit conversions
21	R=1.22 L/D	Optical resolution
22	Scale, area	Saturn ring
23	Scale	New exoplanet
24	V=H x d	Hubble Law
25	Algebra	Visible universe
26	Algebra	Distance and time
27	Size=Angle x Distance	Angular size and resolution
28	Algebra	Angular resolution & temperature
29	L=c(1+z)	Cosmological redshift

I -The Infrared Universe

Goal: Describe how astronomers use infrared light to create different views of objects and learn about their structure and composition.

Crime scene detectives use a variety of technologies to probe a crime scene and put together from clues, what happened. Mass spectrometers can discern the composition of minerals, fluorescent dies can reveal invisible substances and chemicals. Samples of dust and air can reveal traces of left-behind gases. These all benefit from detectives having physical access to samples and testing them in their labs. Astronomers are also trying to understand what happened at the many locations they explore, however, for the vast majority of these, they are too far away for direct contact. Instead, they have to rely on studying the light that objects emit or reflect. Their instruments include imagers that can be tuned to many different wavelengths across the electromagnetic spectrum from gamma rays to radio waves.

Non 'EM' radiations such as particles (cosmic rays, neutrinos) and even space itself (gravity waves) also have important clues to share. Each of these wavelengths of light has something to say about the object that produced them, and these clues, which combined, allow astronomers to learn about the properties and evolution of astronomical objects from near-by planets to the most remote galaxies in the universe.

Figure 1 - Messier 101 in optical (Left: Hubble) and infrared (Right: Spitzer) (Credit NASA/Space Telescope Science Institute)

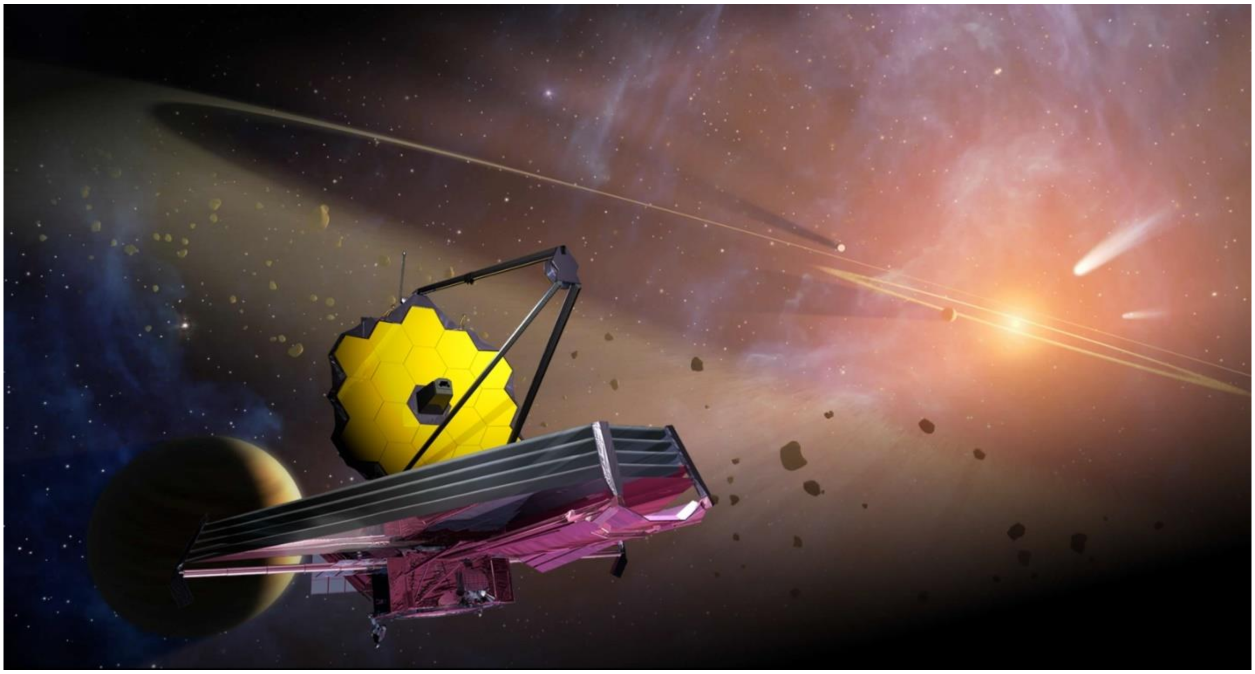


Figure 2 - Artistic rendering of the James Webb Space Telescope, showing the primary mirror and multi-layered sun shield. (Credit: NASA)

Although 'infrared astronomy' had its origin on Earth, our warm atmosphere interferes with detecting the infrared light from most objects in the universe, so astronomers work with NASA and other space agencies to place their specially-designed infrared telescopes into space. Any warm object including the telescope produces its own infrared emission so these telescopes have to be designed so that they are very cold and shaded from direct sunlight in space. Usually this means using cryogens like liquid helium to chill the telescope below 5 kelvins (-268^{o} C) as was the case for the European Infrared Astronomical Satellite (IRAS) launched in 1983, or the Spitzer Space Telescope launched in 2003. Because it is designed for a long operating history, the Webb Space Telescope cannot use active cryogenic cooling because eventually after a few years the liquid helium will evaporate. Instead, it uses a passive-cooling system that consists of an elaborate series of sun shades to keep the telescope in a permanent, frigid temperature of 40 k (-233^{o} C).

So, what can astronomers learn using infrared light? The possibilities are almost endless and rely on a knowledge of how objects emit infrared light. There are two major ways that an object can emit infrared light: Continuum emission and line emission.

Continuum emission - A warm body produces a continuous electromagnetic spectrum that includes energy in the infrared band between 1-micron and 200-microns. This is especially important for detecting interstellar dust grains, which are the building blocks for planets.

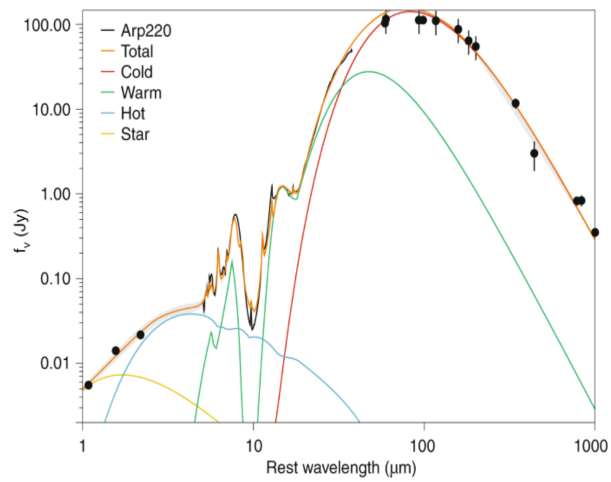


Figure 3 - This is the continuum spectrum of the distant galaxy Arp 220 (brown line) obtained by the Spitzer Space telescope. The infrared measurements between 1 and 1000-microns (dots) are important in distinguishing between the light from various emitting sources such as stars and dust. (Credit L. Armus et al.)

Line emission - Although atoms emit spectral line fingerprints in the visible spectrum between 200 and 800-nanometers, important molecules such as water, carbon dioxide and organic molecules emit their fingerprint lines at infrared wavelengths. This is important if you are trying to detect exoplanets that have atmospheres rich in biogenic molecules.

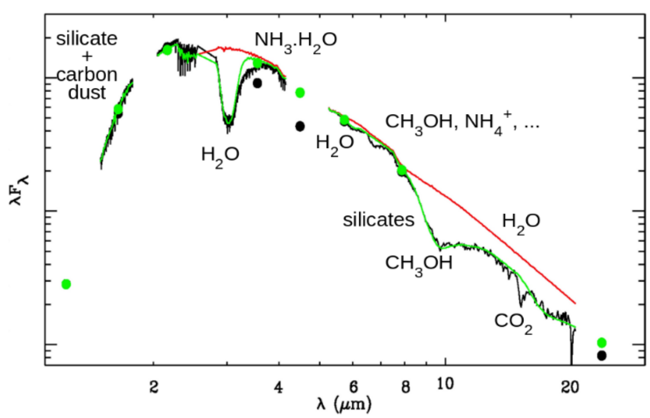


Figure 4 - Infrared spectrum of a highly reddened star tracing silicate and carbon dust with icy mantles in a dense interstellar cloud (black lines). It was observed with the Keck II telescope on Mauna Kea (λ < 4μm) and the Spitzer Space Telescope (λ > 5μm). The red and green lines are models used to extract the dust (red) and ice (green) signatures. (Credit: Institute of Astronomy/U.Hawaii)

Another very important feature of infrared light is that its long wavelengths are better able to penetrate obscuring interstellar dust and so astronomers can see through a dense interstellar cloud to study embedded young stars and planets. For example, here is a pair of images of a dense interstellar cloud called Barnard 68. It is located in the constellation Ophiuchus and is about 400 light years from the sun. It is a small cloud only one-half light years in diameter and contains about as much mass as two of our suns. Figure 5 shows what this cloud looks like at optical wavelengths. It is so dense that it completely obscures the light from more distant stars behind it. It also hides any traces of young stars that may be forming inside it. Figure 6 was taken by the ESO Very Large Telescope at wavelengths between 1 to 2-microns. It shows a nearly transparent cloud with plenty of background stars shining through it, but more importantly no signs of any young stars inside it. Despite its promise as a star-forming interstellar cloud, it has no such activity yet, but it is in the process of collapse so in the next 100,000 years a few infant stars may in fact appear.

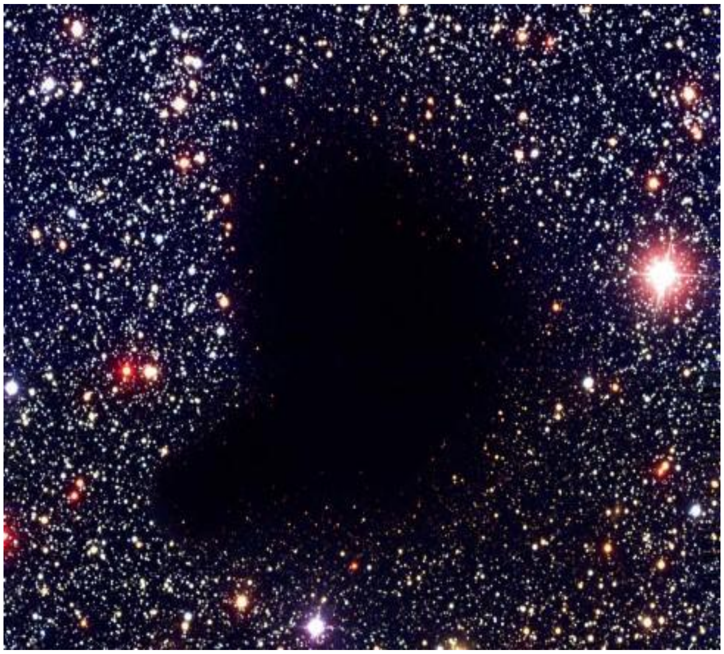


Figure 5 - Barnard 68 in optical light showing dust absorption of background stars (Credit: ESA.)

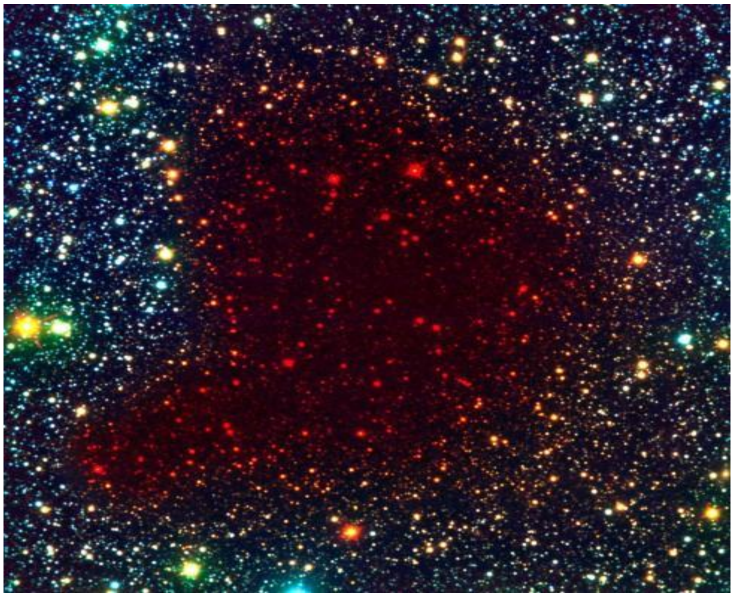


Figure 6 - Barnard 68 in infrared light is nearly transparent to dust absorption revealing background stars. (Credit: ESA / VLT)

Probing the insides of dust clouds to see stars being born is one important activity by astronomers, but discerning the structure of distant objects is also important. Taking advantage of how cool dust grains emit their light at infrared wavelengths, you can disentangle the part of an object’s structure produced by interstellar dust and dust clouds from the much brighter light from stars. This is a critical tool in studying distant nebulae and galaxies to determine how the stars you see are situated with respect to the dust clouds. For nebulae, this gives you insight into where the youngest stars are being formed. For galaxies, it can tell you how active a galaxy is in creating new stars and in which part of the galaxy star formation is occurring. Here are some examples of images taken in the infrared of specific kinds of objects, which help astronomers uncover their structure and properties.

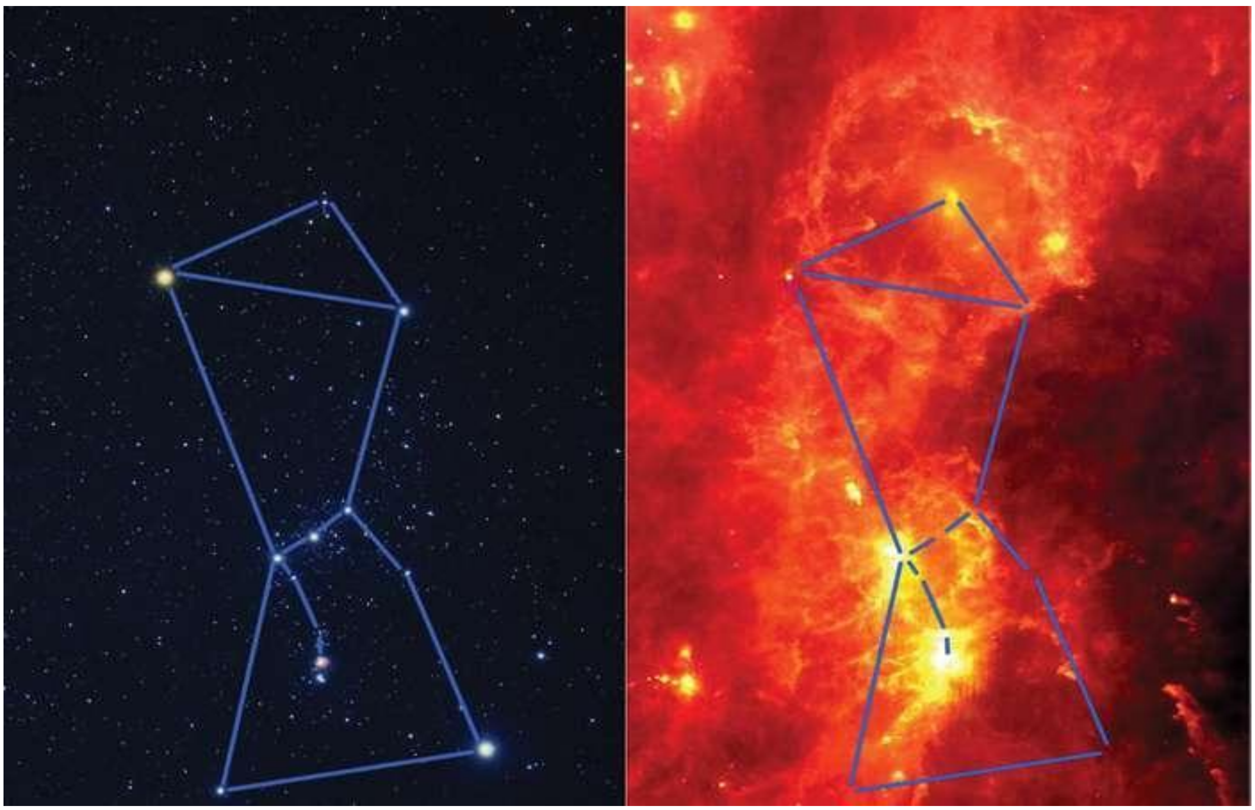


Figure 7 - The constellation of Orion viewed with optical light (left: Credit Akira Fujii) and infrared light (right) taken with the IRAS telescope (Credit: ESA/IRAS).

The stars in the constellation Orion are very hot and no more than a few million years old. They are capable of heating up dust grains to distances of many light years making them shine brightly at infrared wavelengths. These hundred or so stars are at the front of a vast interstellar cloud from which they formed, and are now tunneling their way out to produce the Orion Nebula and others surrounding the constellation. Before the advent of infrared imaging, astronomers could only study this cloud from glimpses surrounding the Orion Nebula, but now it can be seen by the light of its warm dust grains and mapped across a huge swath of the sky as viewed from Earth.

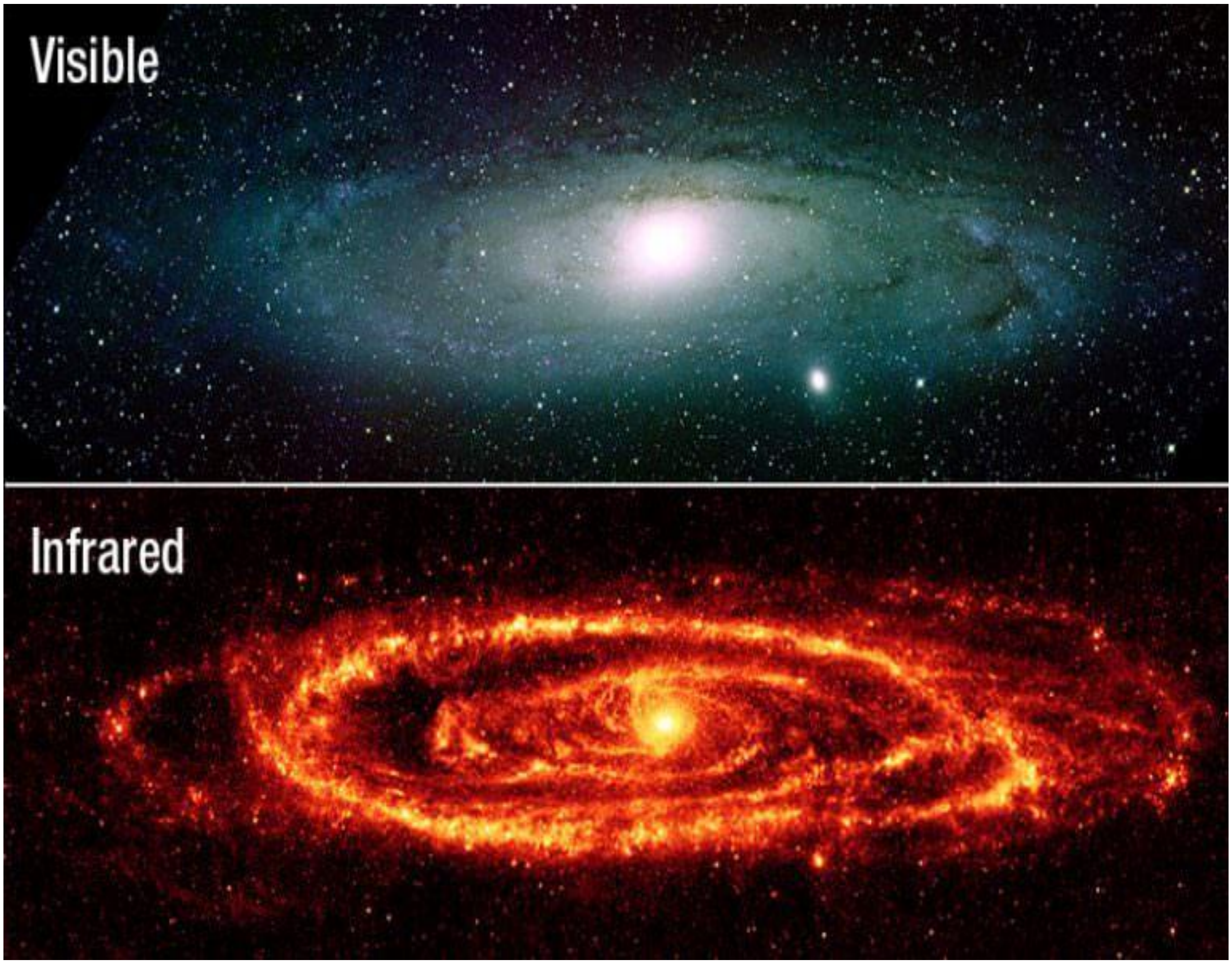


Figure 8 - Two views of the Andromeda Galaxy (Messier 31) Top) Optical image (Credit: Herschel ATLAS/ESA) Bottom) Spitzer Space Telescope view at a wavelength of 24-microns (Credit: K. Gordon/JPL/NASA)

The Andromeda Galaxy is a near-twin to our Milky Way. For a century of telescopic study, its proximity of only 2.5 million light years has given astronomers a view of what a spiral galaxy looks like. In fact, if human eyes were sensitive enough, it would span an area of the sky about six times the diameter of the full moon. But even the most detailed, optical studies showed it to be a tantalizing, but distorted, spiral galaxy with dense spiral arms and a multitude of obscuring dark clouds that hid its true shape. With infrared light, the confusing starlight in the galaxy can be suppressed and only the warm dust grains revealed. This infrared view shows a complex interstellar medium filled with numerous star-forming clouds strung out in distinct spiral patterns.

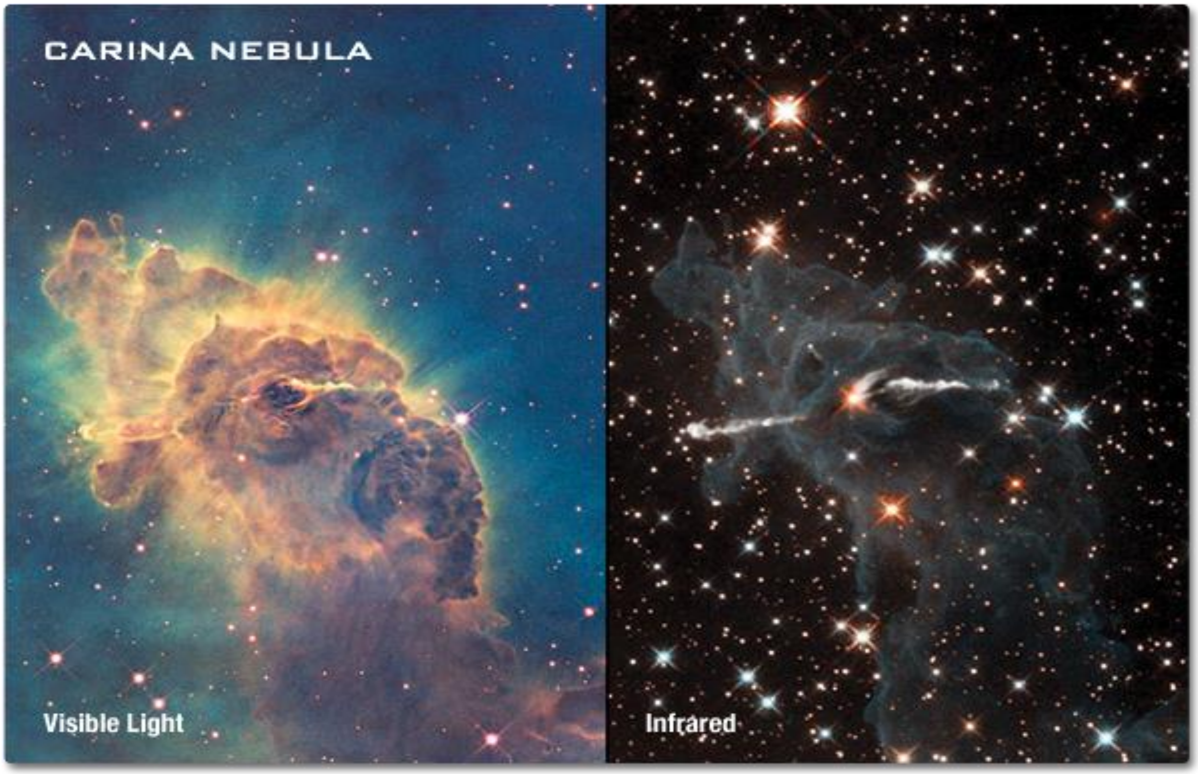


Figure 9 - Two views of the Carina Nebula (Credit: NASA, ESA, and the Hubble SM4 ERO Team*)*

Stars like our sun were born in interstellar clouds but so deeply embedded astronomers cannot study this process very well in the nurseries that are close to us in space. Optical images often show detailed cloud shapes but only let us see the surface layers of this process far from the forming star. With infrared imaging, these dense obscuring clouds can be penetrated to reveal the hidden structures within. In the case of the Carina Nebula, a star-forming site is mid-way between two jets of matter traveling in opposite directions. Astronomers can now study how stars like our sun are formed, and the role that is played by these jets.

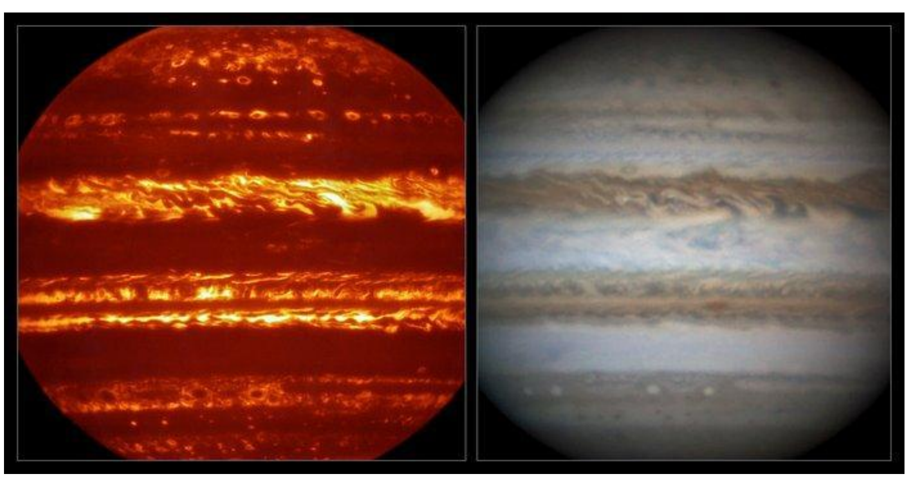


Figure 10 - This view compares a view of Jupiter from VISIR (left) at infrared wavelengths with a very sharp amateur image in visible light from about the same time (right). (Credit: ESO/L.N. Fletcher/Damian Peach).

Taking advantage of how infrared light tracks the location of heat sources in distant objects, astronomers can turn these telescopes on the planets in our solar system to investigate how their internal heat energy flows out of the planet. In this case, Jupiter's clouds can now be seen as they are heated by the planet's interior and faintly glow. The atmosphere is not uniformly warm but is filled by convecting clouds that dredge warm material from deeper inside the planet and bring it to the cloud-tops to form the infrared image.

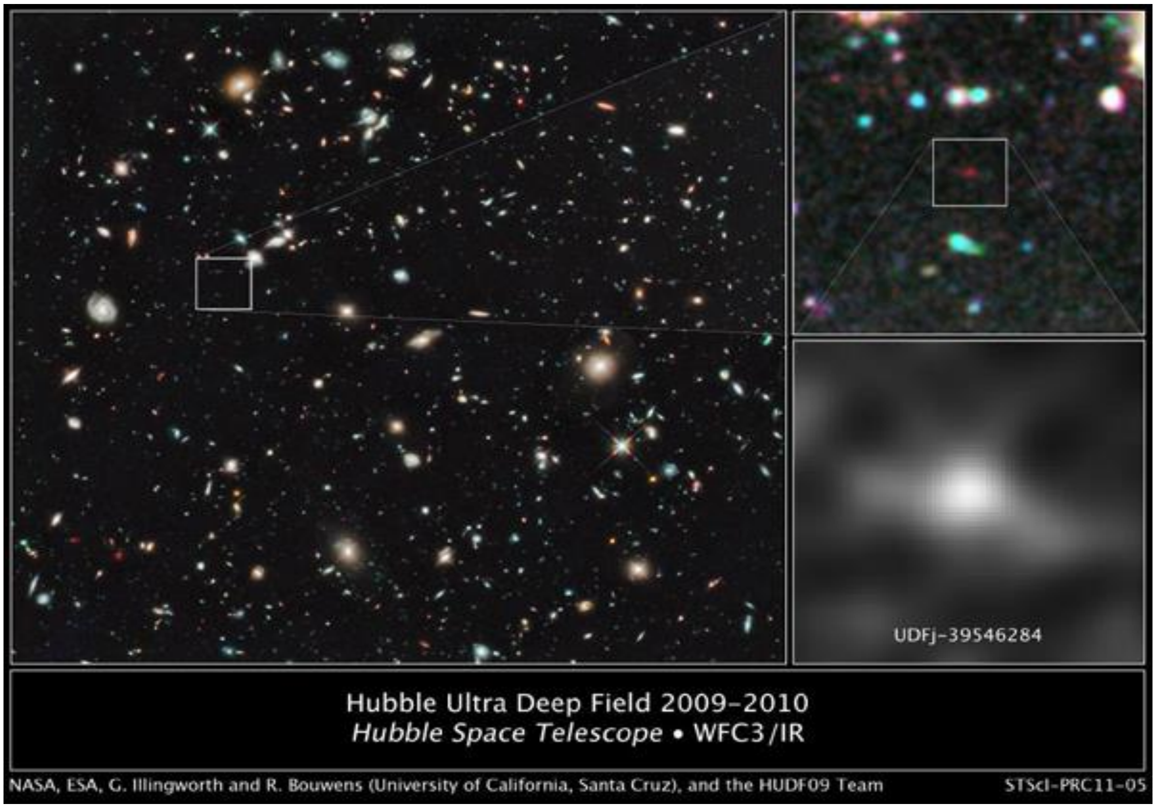


Figure 11 – A portion of the Hubble Deep Field showing numerous galaxies (Credit: NASA/HST)

Even though Hubble was not designed to see infrared light, its 'red' filter picks up the faint light from high-reddened, young galaxies such as this view of UDFj-39556284. These galaxies are important because they are among the youngest galaxies formed in our universe 450 million years ago. Because our universe is expanding, the space between galaxies is stretching and so the wavelengths of any light rays also get stretched. When a galaxy is first formed, its young luminous stars shine brightly at ultraviolet wavelengths, but by the time this light arrives at Earth 13 billion years later its wavelengths have been stretched so that they are now measured in microns and not nanometers. This means that the youngest galaxies in the early history of the universe will appear to us today as very dim, red objects shining brightly in the infrared spectrum. The Hubble Space Telescope cannot see these very well, but the much-larger Webb Space Telescope will clearly see them in the infrared and be able to take high-resolution images of many of them.

For the JWST, the scientific mission starts about six months after launch when all of the sensor calibration and engineering check-outs of the telescope have been completed. This First Light event will probably occur sometime in late-Spring 2022. The observing time on the telescope has already been scheduled, and data from these investigations will start being reported to the public almost immediately. The telescope has been designed with four specific goals in mind, as shown in the table below.

- **Provide clear images of the earliest stars and galaxies to form in our universe from a time about 100 million years after the Big Bang.**
- **Provide imagery and spectroscopic data on nearby interstellar clouds, nebulae and nearby galaxies in the universe to help astronomers develop a better understanding of their origins.**
- **Investigate exoplanets orbiting nearby stars to determine their temperatures, surface temperature maps, and atmospheric chemistry in the continued search for planets capable of sustaining life.**
- **Study the various objects orbiting the sun in our own solar system to discover new dwarf planets, comets and asteroids, and to investigate their chemistry.**

The Hubble Space Telescope was not able to carry out most of these activities because it was not designed to detect infrared light, which is the wavelength range in which most of the JWST objects emit their energy. Other infrared telescopes such as the Infrared Astronomical Satellite (IRAS), the Wide-field Infrared Survey Explorer (WISE) and the Spitzer Space Telescope had much smaller mirrors and so were less capable of detecting the farthest, coolest and dimmest objects especially the youngest (most distant) galaxies, which emit no optical energy as viewed from Earth.

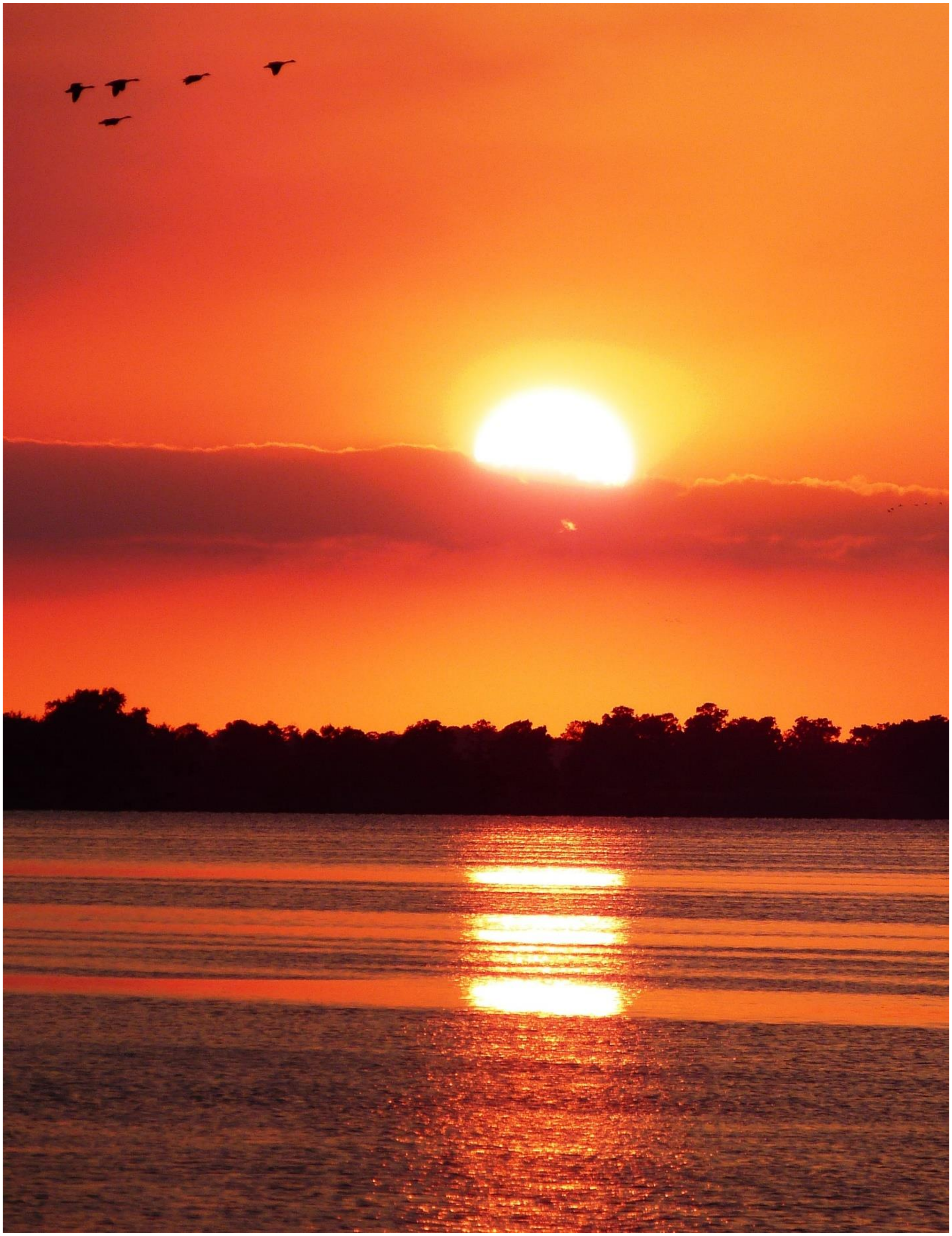


Figure 12 – A sunset is a rich source of reddened electromagnetic waves in the visible spectrum. (Credit Wikipedia/USFWS/Allie Stewart Creative Commons Attribution 2.0 Generic license).

II -The Electromagnetic Spectrum

Goal: Present the electromagnetic spectrum and show where infrared light is located.

Light is a form of energy in which a changing electric field creates a changing magnetic field, and both of these travel through space at a speed of 300,000 km/s - called the speed of light. James Clerk Maxwell predicted the existence of these ‘electromagnetic waves’, and in Heinrich Hertz in 1887 confirmed their existence in his discovery of radio waves. He actually created electromagnetic waves by creating an oscillating transmitter that produced a periodic spark, and with another loop of wire he detected the synchronous waves from this spark in a second instrument. When he found that the speed of the transmission was the same as Maxwell had predicted for electromagnetic waves, this confirmed that they could be made artificially and had the expected properties.

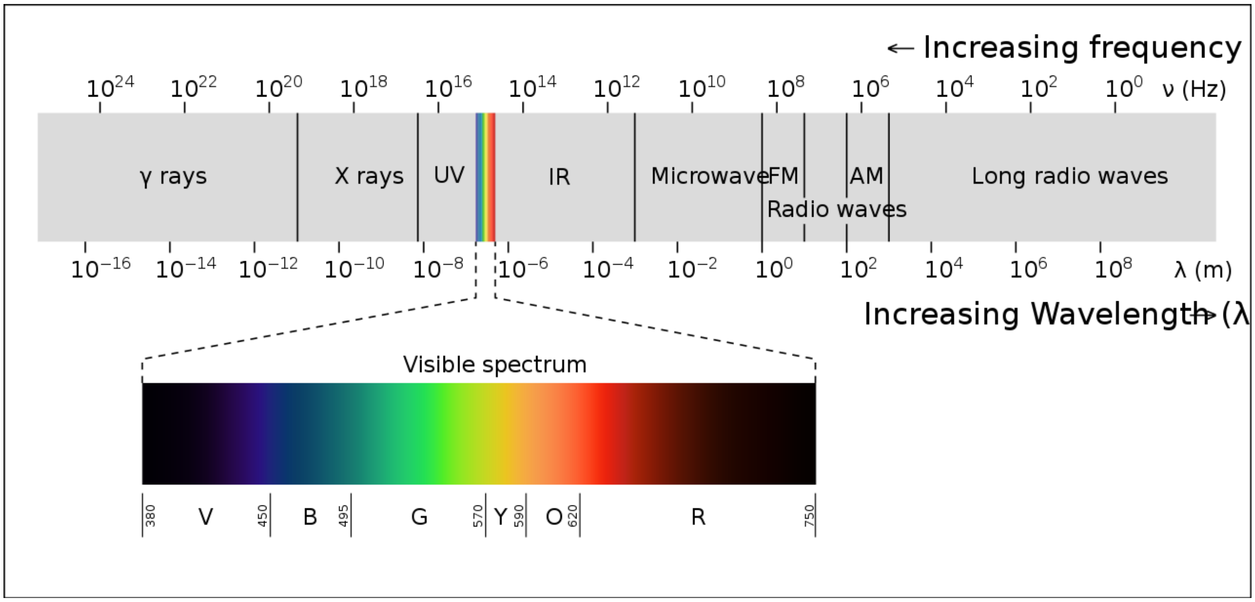


Figure 13 - The basic parts of the electromagnetic spectrum and their common names. (Credit: Wikipedia/Philip Ronan - Creative Commons Attribution-Share Alike 3.0 Unported license.)

Electromagnetic waves, and in fact waves of all kinds including sound waves and seismic waves, follow a very simple relationship between their speed, frequency and wavelength which is in words expressed as speed = frequency x wavelength. It can also be written mathematically as $V = \lambda \nu$ where V is the speed of the signal in meters/sec, λ is the wavelength of the wave in meters, and ν is the frequency of the wave in cycles-per-second (called a Hertz). For sound waves at sea-level, V is the speed-of-sound of about 340 m/s. For electromagnetic waves we use the symbol c for the speed-of-light, which is 300,000,000 meters/sec.

Problem 1 – Wavelength and frequency

Unlike sound waves that need the air to travel through, electromagnetic waves do not need a medium to carry them. This was the big challenge to understand during the 19th century as physicists searched in vain for an 'ether' to transport electromagnetic (EM) waves. EM waves are produced by changes in the electric field surrounding a charged body, and these fields reach out into space for great distances. A disturbance in this electric field causes a magnetic field to be created simultaneously, and this pair of waves moves through the electric and magnetic fields as a combined electro-magnetic wave.

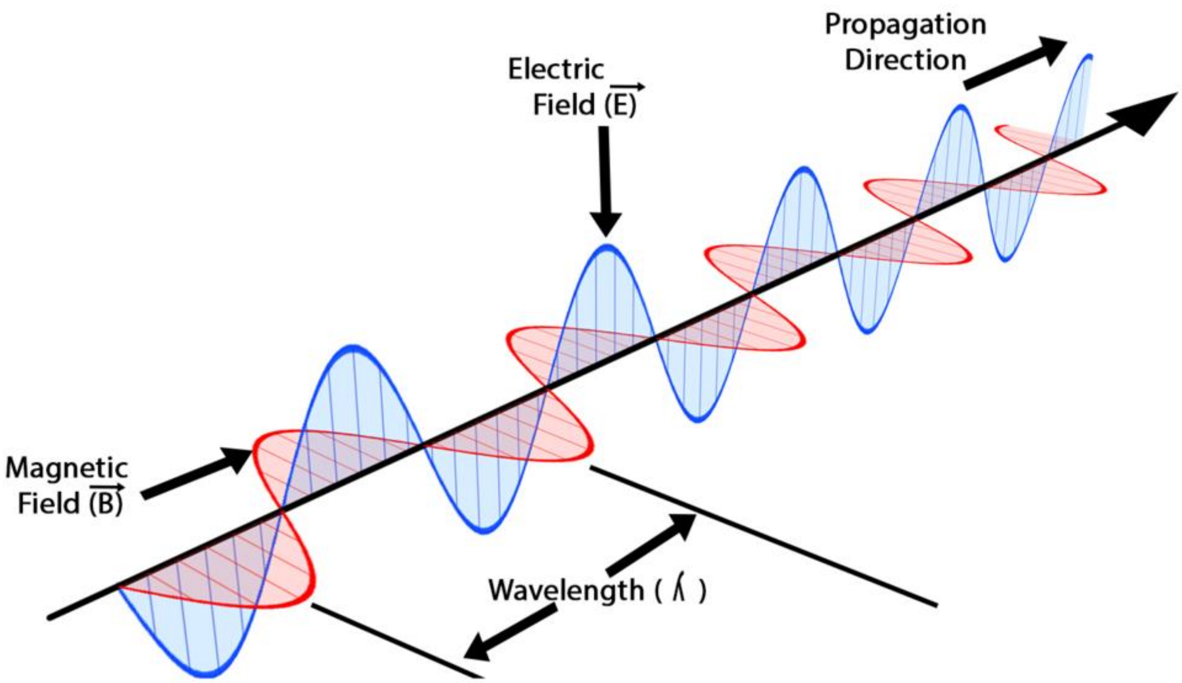


Figure 14 - An example of an electromagnetic wave, which also defines its wavelength (λ). (Credit: Wikimedia/Dechammakl - Creative Commons Attribution-Share Alike 4.0 International license.)

A-Wavelengths made simple – working with micron units

Goal: Present the scale of infrared wavelengths in micron units and compare to the sizes of small objects

In infrared astronomy, the wavelengths for this form of electromagnetic wave are measured in very small units called microns. We use these units because they are far more compact to state in written text than the equivalent SI unit of meters. For example, at near-infrared wavelengths, these are measured as 0.000001 meters or 10^{-6} meters, but you can also write them as 1-micron (abbreviated as μm). The only time astronomers actually use the value in meters is when they are plugging the number into a formula to calculate some other property of the source, however, some formulae can be written for convenience in such a way that you can use the value in microns directly. Microns are very small units, but we come into daily contact with devices that broadcast at these wavelengths.

Table 3 – Examples of objects emitting infrared energy

Cellphone	160,000 microns	Security screening	300 microns
Microwave oven	122,000 microns	COBE/DIRBE	240 microns
Police radar	8,600 microns	IRAS longest	100 microns
Microwave comm.	3,750	Webb longest	28 microns

Table 4 – Examples of sizes of different objects

- Amoeba……………………………………… 500 microns
- Size of a period……………………………. 400 microns
- Grain of salt…………………………………. 300 microns
- Beach sand………………………………….. 90 microns
- Human hair ………………………………….. 70 microns
- Ephiphytic orchid seed…………………….. 50 microns
- Grass Pollen……………………………………25 microns
- Cat down hair…………………………………. 17 microns
- White blood cell………………………………12 microns
- Soot ……………………………………………….5 microns

Problem 2 – Working with scale models to explore micron units

Figure 15 is an electron microscope view of a simple micro-electro-mechanical (MEM) device called a microcantilever. It is a thin hair-like rod that is fixed at one point and free to vibrate (as indicated by the blur on its upper end). This causes a slight voltage change that is measured to determine the frequency of the sensor. The width of the two vertical white lines is 17.6-microns.

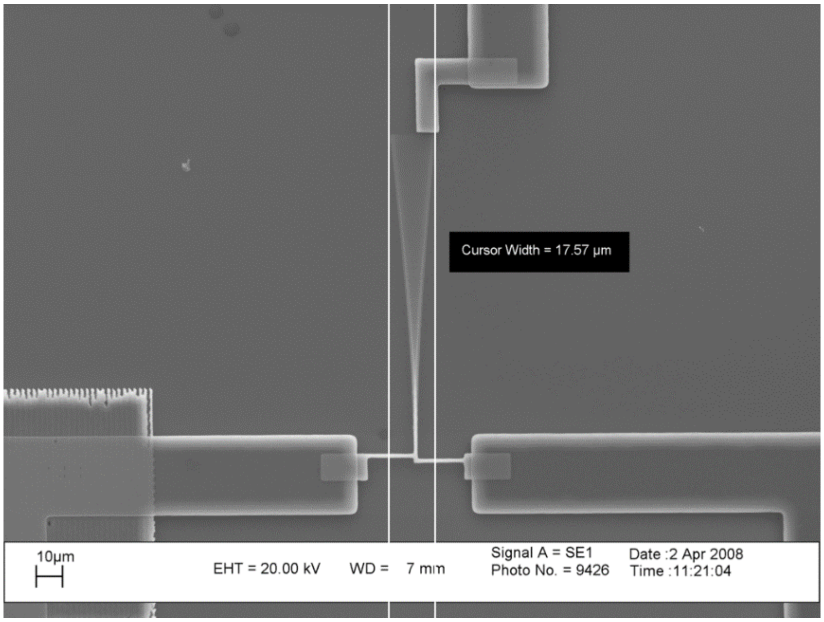


Figure 15 – An electron microscope image of a MEMs resonator device (Credit: Wikipedia/ Patrick Fletcher - Creative Commons Attribution-Share Alike 3.0 Unported license)

B-Kelvin Temperatures and Very Cold Things

Goal: Kelvin temperature units and how to convert to Fahrenheit and Celsius.

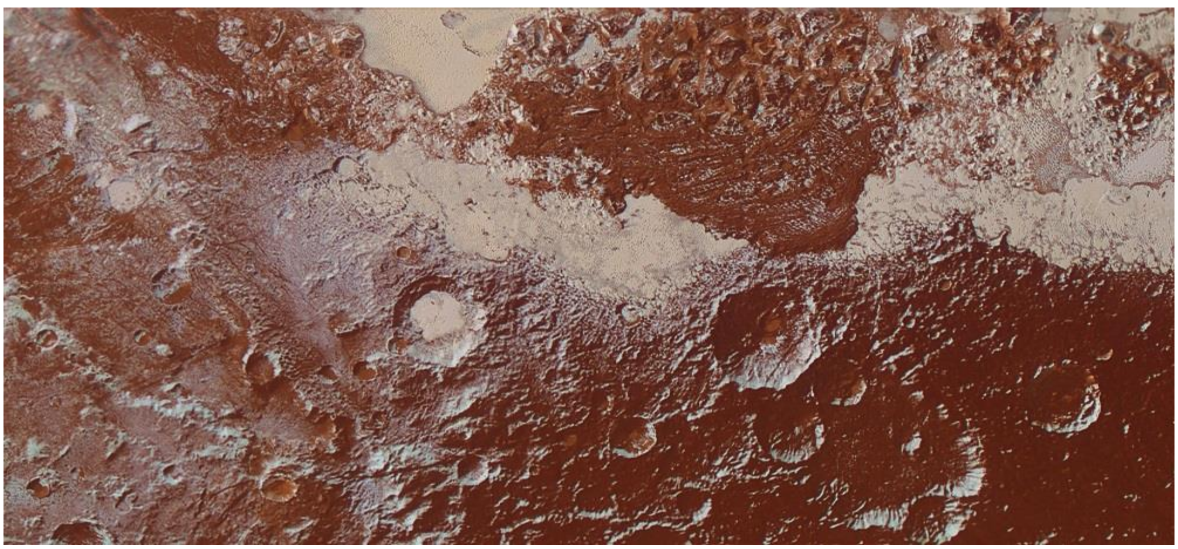


Figure 16 - The surface of Pluto is one of the coldest places in our solar system at a frigid 33 kelvins in the winter. (Credit: NASA/New Horizons).

To keep track of some of the coldest things in the universe, scientists use the kelvin temperature scale which begins at 0 kelvin, which is also called Absolute Zero. Nothing can ever be colder than Absolute Zero because at this temperature, all motion stops. The table provided in Problem 3 shows some typical temperatures of different systems in the universe. You are probably already familiar with the Celsius (C) and Fahrenheit (F) temperature scales.

Problem 3- Kelvin Temperatures and Very Cold Things

C-Exploring Black body curves

Goal: Students learn about the black body curve and its important features

When a body heats up, the electromagnetic radiation is at infrared wavelengths, but the intensity of these waves is not random; they follow a very precise mathematical curve that was first predicted by Max Planck in 1900. These curves are called Planck curves, or Planck distributions. They are also called black body curves because if an object absorbs all of the radiation that falls on it (perfectly black) it will emit electromagnetic radiation that follows the Planck curve, which has the following parts:

- Peak emission – This is where the curve has its highest value in brightness or intensity.
- Short wavelength cut-on - On the short wavelength side of the peak, this is where the radiation from the body would have its lowest or zero brightness value. Very cool objects emit light in the infrared and are invisible in the optical wavelength band.
- Long wavelength tail – This is the part of the curve at long wavelengths where the brightness is decreasing as the wavelength increases. The object will appear dimmer and dimmer as the wavelength increases.

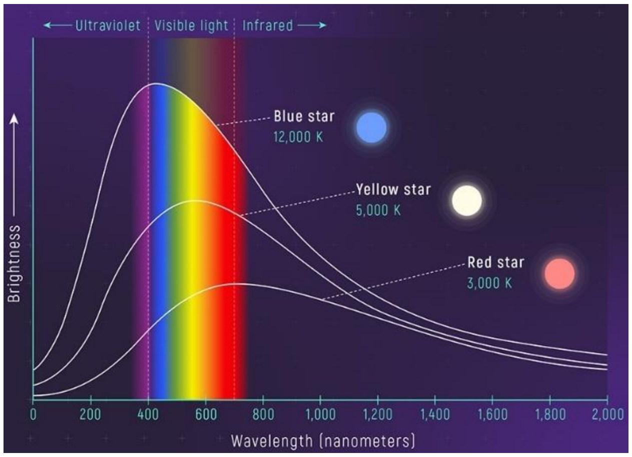


Figure 17 – Stars at different temperatures have different colors according to the wavelength where most of the light is emitted according to the black body curve at that temperature. Given the temperature of the object, the shape of the curve is used to predict the brightness of the object at different wavelengths. (Credit: NASA, ESA, Leah Hustak, STScI)

D-Exploring Wien's Displacement Law: Temperature and wavelength

Goal: Students learn about how temperature is related to the wavelength where an object emits most of its light using a simple formula.

$$\lambda = \frac{2898}{T}$$

Black body curves can be used to show why it is that as temperature increases, the color of an iron bar shifts in color. This is a well-known phenomenon observed by blacksmiths, and you can even see this yourself if you have an electric hot plate with a temperature dial. This phenomenon is related to the way that the peak of the black body curve shifts in wavelength as you change the temperature of the object emitting the light. A simple formula shown below lets you calculate the peak wavelength in microns (λ) given the temperature in kelvins (T).

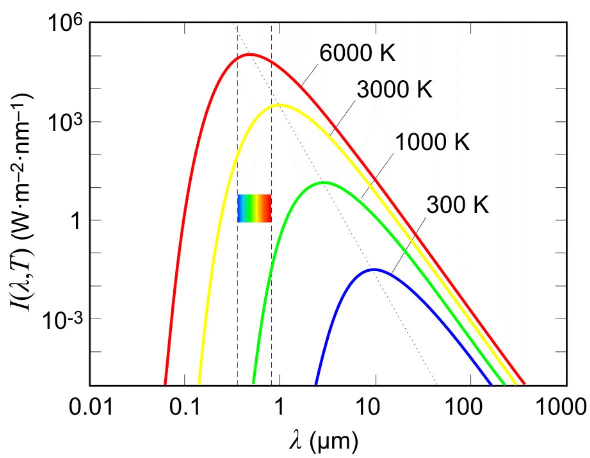


Figure 20 – A series of Planck curves of different temperature. (Credit Wikipedia / High Templar)

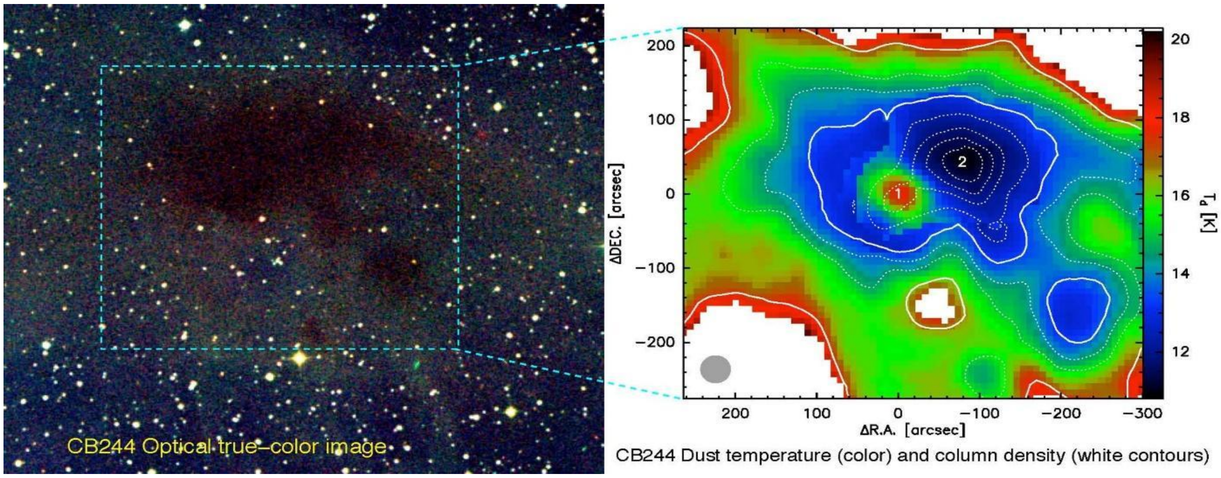


Figure 21 – An infrared map of the dark nebula CB244. (Credit: ESA/Herschel)

Problem 4 - Understanding the Wein Displacement Law.

Problem 5 - Why are hot things red?

E-Exploring the Stefan-Boltzmann Law

Goal: Students learn about the relationship between temperature and luminosity

$$\boldsymbol{F = \sigma T^4}$$

The Planck curve gives the brightness of an object at a specific temperature in terms of the amount of energy that is emitted by a square-meter of surface every second and over a specific range of wavelengths. You can think of this is a bar graph where the height of the bar is the intensity in watts per square meter, and the width of the bar is a small unit of wavelength. But what would happen if you added up all of the bars across the entire electromagnetic spectrum and found the area under the Planck curve? What you would get would be the power (called its luminosity) of electromagnetic radiation emitted by the object. This would include the radiated power across the visible spectrum as well as all of the power emitted at all other wavelengths. This calculation can be performed using the methods of integral calculus, but the result is something remarkable first discovered by Josef Stefan in 1879 and Ludwig Boltzmann in 1884 called the Stefan-Boltzmann Law. What it says is that the total power emitted by an object over a square-meter of its surface (called its flux: F) is proportional to the fourth-power of the surface temperature in kelvins. The 'constant of proportionality' is called the Stefan-Boltzmann Constant and has a value of σ=5.67x10^{-8} watts/m^2/k^4.

For example, our sun has a surface temperature of 5770 k, so each square meter of its surface emits about $6.3x10^7$ watts of EM energy. From the previous discussion of the Wien's Displacement Law, the peak of this energy comes out at a wavelength of (2898/5770) = 0.5 microns, which is at the middle of the visible spectrum where the human eye is the most sensitive.

Figure 22 – An example of how area and temperature are related to heat output in a common fire. (Credit: Wikipedia: (match flame) Creative Commons Attribution-Share Alike 4.0 International license. (camp fire) Einar Helland Berger - Creative Commons Attribution-Share Alike 2.5 Generic license).

A more common example is for the case of an ordinary fire. In Figure 22 we see a match and a camp fire. Both are burning at a temperature of 873 k and from the Stefan-Boltzmann produce an amount of power per square meter of $F = 5.67x10^{-8}$ x $(873)^2$ = 33,000 watts/meter^2. The peak of this flame's emission is at a wavelength of (2898/873) = 3.3-microns. So why is it that the camp fire is dramatically hotter to someone standing near it than a candle? The answer is in the total surface area that is emitting the infrared energy.

F- Flux and Luminosity: A matter of areas.

Goal: Students learn about how luminosity, temperature and area are related

The Stefan-Boltzmann Law lets us calculate from the temperature of a surface the amount of power it is emitting onto space for each square-meter of its surface. For many applications in astronomy, we want to know the total amount of electromagnetic energy being released by the distant object over its entire surface area. This power, in watts, is called luminosity. To calculate it, you just need the temperature of the surface and the radius, R, of the object if it is a sphere like a star. For a spherical body, its surface area is just A = 4π R^2, so the luminosity of a spherical body is just

$$L = 4\pi\sigma R^2 T^4$$

Let's see how this formula works for a star like our own sun. For T=5770 k and R = 6.96×10^8 meters we get a surface area A = 6.1×10^{18} m^2. Then from the Stefan-Boltzmann Law we get F= 6.3×10^7 watts/m^2. The product of F x A is then L=3.8×10^{26} watts.

When a star expands and cools to become a red giant, its temperature can be a chilly 3,500 k (hence the name 'red') but be about 200 times larger than our sun. The luminosities are then about L = 2.1×10^{30} watts which is about 5,500 times as luminous as our sun, which is why they are called red giant stars. Our sun will eventually become one of these in another 5 billion years.

Problem 6 - Stellar Temperature, Size and Power

Problem 7 - Exploring a Dusty Young Star

Problem 8 - Deriving the Stefan-Boltzmann Constant

G-Night vision vs thermal imaging

Goal: Students learn the difference between night vision and thermal imaging

You have probably heard about special goggles that let you see in the dark. This technology was originally developed for military applications but in the last few decades this technology has become widely available to the private citizen. In fact, there are apps you can download that also provide this ability in a limited way. This 'night vision' technology amplifies the faint light reflected from an object such as a person or animal in a dark or twilight forest and lets you see a brighter picture of it. Night vision needs some form of ambient faint light to illuminate an object in order for you to see its image. Night vision is now available in some recent-model cars to help with dim light conditions such as unlit highways and tunnels.

Figure 23 – An example of a night vision scene observed through an image-intensifier (Credit: Wikipedia/ US Army)

Night vision needs nearby visible light to work properly. Thermal imaging, on the other hand, does not need any light to function. Night vision works by amplifying nearby visible light. Thermal imaging works by using infrared sensors to detect differences in temperatures of objects in its line of sight by directly detecting their infrared light.

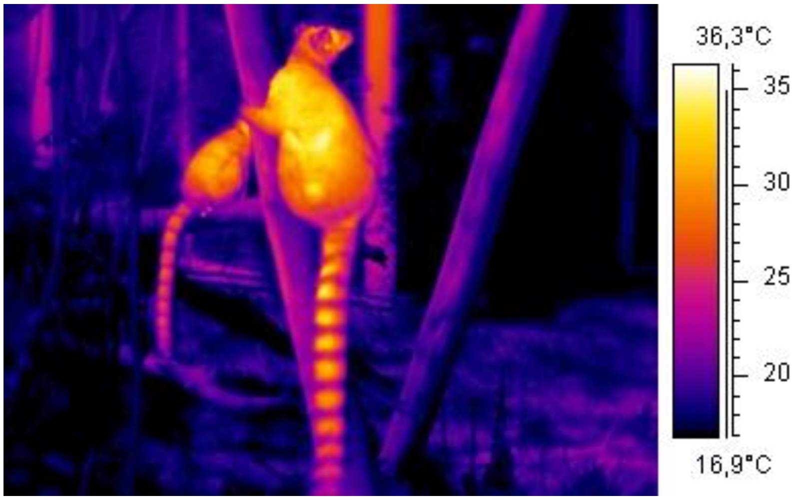


Figure 24 – A thermographic image of a lemur in a tree at night, observed by its heat emission. (Credit: Wikipedia; Arno / Coen - Creative Commons Attribution-Share Alike 3.0 Unported license).

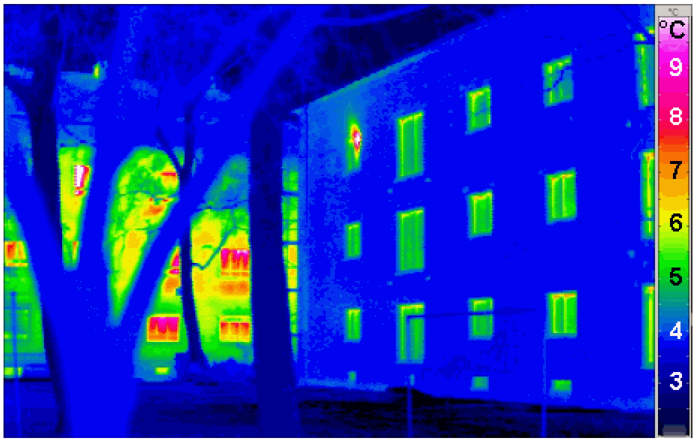


Figure 25 – A thermal imaging scan of a house showing hot and cool areas. Hot areas require more insulation to lower the leakage of heat from the interior. (Credit: Wikipedia; Passivhaus Institute - Creative Commons Attribution-Share Alike 3.0 Unported license)

An infrared camera can measure the brightness of a house at several wavelengths and create a temperature map showing where the house is warmest and coolest. The temperatures are color-coded to make them easier to read than a map of numbers. Blue colors often mean cooler temperatures and less heat loss, red usually means hottest temperatures and the greatest heat loss. By measuring the infrared light from distant objects, astronomers use thermal imaging to detect very cold objects that cannot be easily seen, or may even be invisible, at optical wavelengths.

Problem 9 – A home energy audit via infrared emission.

H– How filters work

Goal: Students learn how filters work to block some wavelengths and allow others to pass.

A filter blocks the light from wavelengths that you are not interested in seeing or detecting with your camera. For example, a pair of sunglasses is an example of what is called a broad-band filter because it reduces the intensity of light over a wide range of visible light wavelengths. Other kinds of filters only pass light across a narrower band of wavelengths. For example, the human eye's retina has cone receptors that are most sensitive to red, green and blue light wavelengths in the optical spectrum. Smartphone cameras also use 'RGB' filters to take separate images that can then be combined to create a color picture.

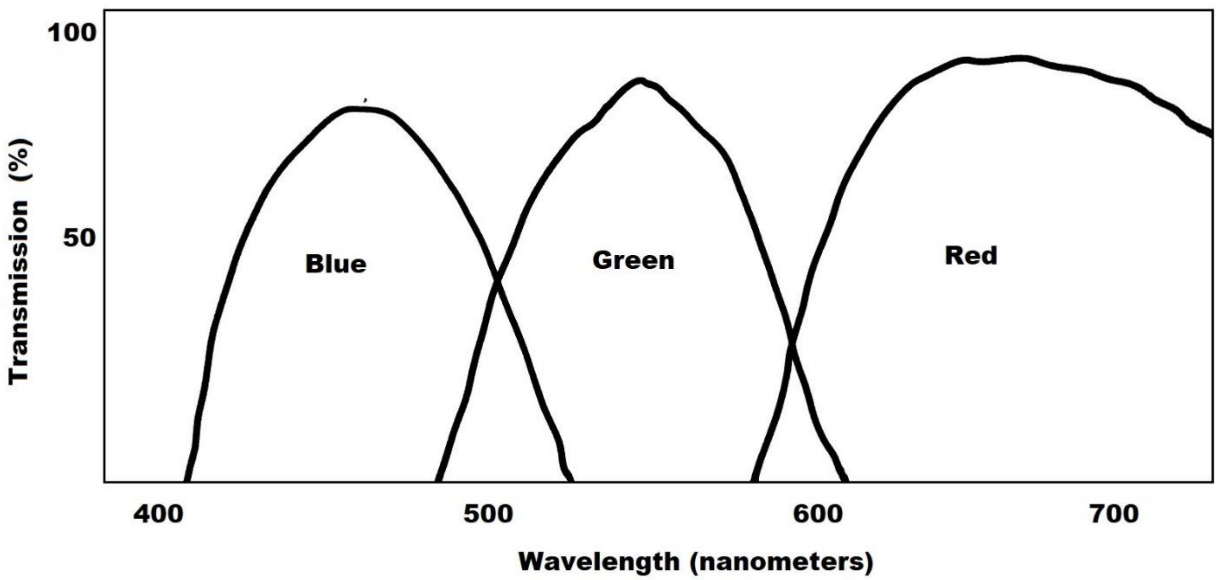


Figure 26 – Typical examples of the wavelength transmission of three common 'RGB' filters. (Credit: Sten Odenwald)

Problem 10 – Understanding filters.

Filters are not perfect. Due to the technology used to create them, some can have 'leaks' that let undesired wavelengths of light pass through and be recorded. The R filter can often have long-wavelength leakage that extends into the infrared region of the spectrum at wavelengths longer than 700 nanometers (0.7 microns).

The curves in Figure 27 show the sensitivity of the human eye to visible light from the three light-sensing retinal elements corresponding to RGB.

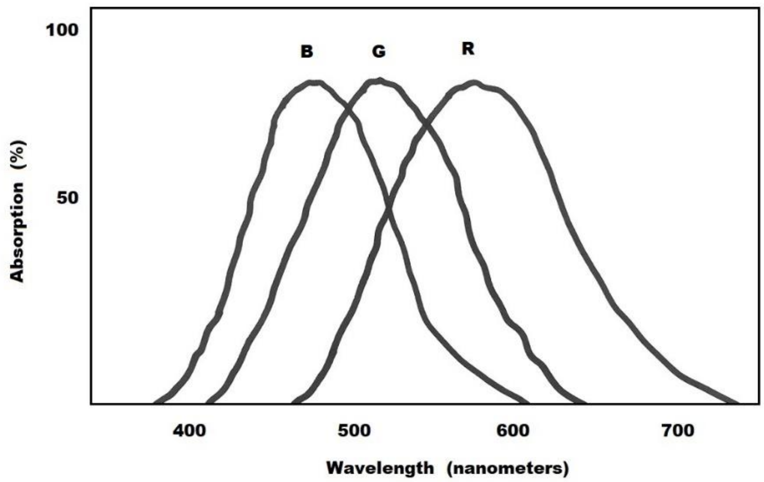


Figure 27 - Example of the sensitivity curves for the three types of cones in the human retina. (Credit: Sten Odenwald)

Note that the human R band (red line) has a sharper cutoff at wavelengths longer than 650 nanometers. This means your camera's R filter shown in Figure 26 admits more light at wavelengths longer than 650 nanometers and so it will be able to see this long-wave light while your eye may see nothing at all!

Infrared telescopes are often designed to work within specific wavelength bands. For the Webb Space Telescope, its sensors and imagers operate in three bands. The Near-Infrared Camera (NIRCam) covers wavelengths between 0.6 and 5-microns, the Near-Infrared Imager and Slitless Spectrograph (NIRISS) covers a similar waveband from 0.8 to 5-microns, and the Mid-Infrared Imager (MIRI) covers 5.6 to 25.5-microns. The two near-infrared instruments are designed to detect and map warm dust surrounding young forming stars and to detect molecules in the atmospheres of exoplanets, while the mid-infrared MIRI instrument will create images of very cool objects and young galaxies that formed just after the Big Bang. With its coronagraph, it will also block out the light from nearby stars and search for the much fainter light from orbiting exoplanets.

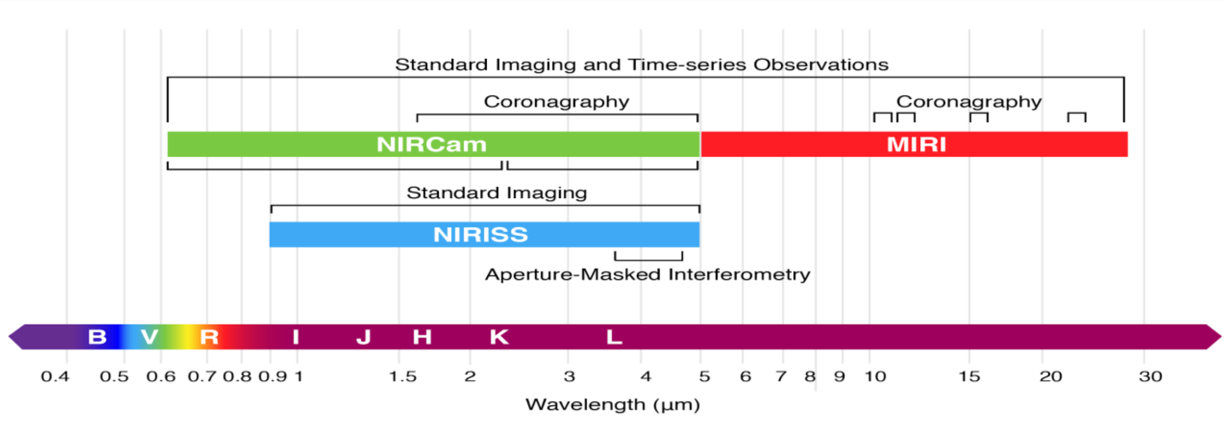


Figure 28 – The wavelength locations of the main instruments on the Webb Space Telescope (Credit JWST/NASA)

Problem 11 - Working with filters

I- Non-contact thermometers

Goal: Students learn about medical thermometers that do not require skin contact as examples of measuring infrared light to determine temperatures of remote objects.

Figure 29 – Example of a typical digital infrared thermometer.

Most medical thermometers use alcohol as a fluid in a capillary tube that expands along a calibrated scale to show temperature. Modern-day digital thermometers use a sensor called a thermocouple, which generates a small current between two dissimilar metals in contact. Other thermometer devices use a thermochromic or liquid crystal material that changes color in specific temperature ranges. All of these require direct contact with the material to be measured and the heat energy passes from the skin directly into the material that is serving to measure the heat. A newer technology called non-contact thermometers do not require direct contact and are therefore medically safer to use because they do not have to be washed or disposed after a patient has been measured.

Non-contact thermometers such as the one in Figure 29, work by detecting the infrared radiation emitted by the heated surface. This means that, unlike conventional thermometers, they cannot measure the temperature inside an object, which could be a problem for some medical diagnoses. They can measure temperatures in the range from -60^o F to +1000^o F (222 k to 311 k) at an accuracy of 0.1^o F, and do so in less than a half-second.

III - Spectroscopy: Discovering the fingerprints of the cosmos

Goal: Students learn how spectroscopy is used to determine the chemistry of distant objects

Once black body radiation and its relationship to temperature was discovered, astronomers were able to determine the temperatures of any astronomical object by merely measuring its spectrum and matching it with a Planck curve of the appropriate temperature. This set the stage for classifying stars, not only by their luminosity (wattage) but by their temperature (red giant, white dwarf). From this, astronomers eventually determined how stars of different temperatures and luminosities were related to how they evolved in time. But light can be a conveyor of far more information than just temperature.

Beginning with the advent of laboratory spectroscopy in the mid-1800s, astronomers soon adapted this technology to determine the element compositions of the sun, planets, stars, nebulae and galaxies over the course of the next century. The Lick Observatory's Stellar Spectrometer was the early-20th century workhorse for measuring individual stellar spectra. Photographic techniques were used by Harvard astronomers Annie Cannon and Edward Pickering to classify hundreds of spectra simultaneously using a simple prismatic wedge fitted over the aperture of the telescope to disperse the starlight.

These spectro-photographic technologies grew in importance in the early-20th century as astronomers such as Vesto Slipher, who was the first to detect galaxy doppler shifts with enough clarity to measure them, and Edwin Hubble turned their attentions towards faint galaxies and discovered the expansion of the universe. Also, George Ellery Hale in 1895 discovered that if you used the spectroscope as a filter, you could scan the light from individual spectral lines and build up an image of the sun at specific wavelengths. This ushered in a technique called spectroheliometry, which is widely used today, not just for studying solar activity, but to examine the multi-wavelength properties of nebulae and galaxies.

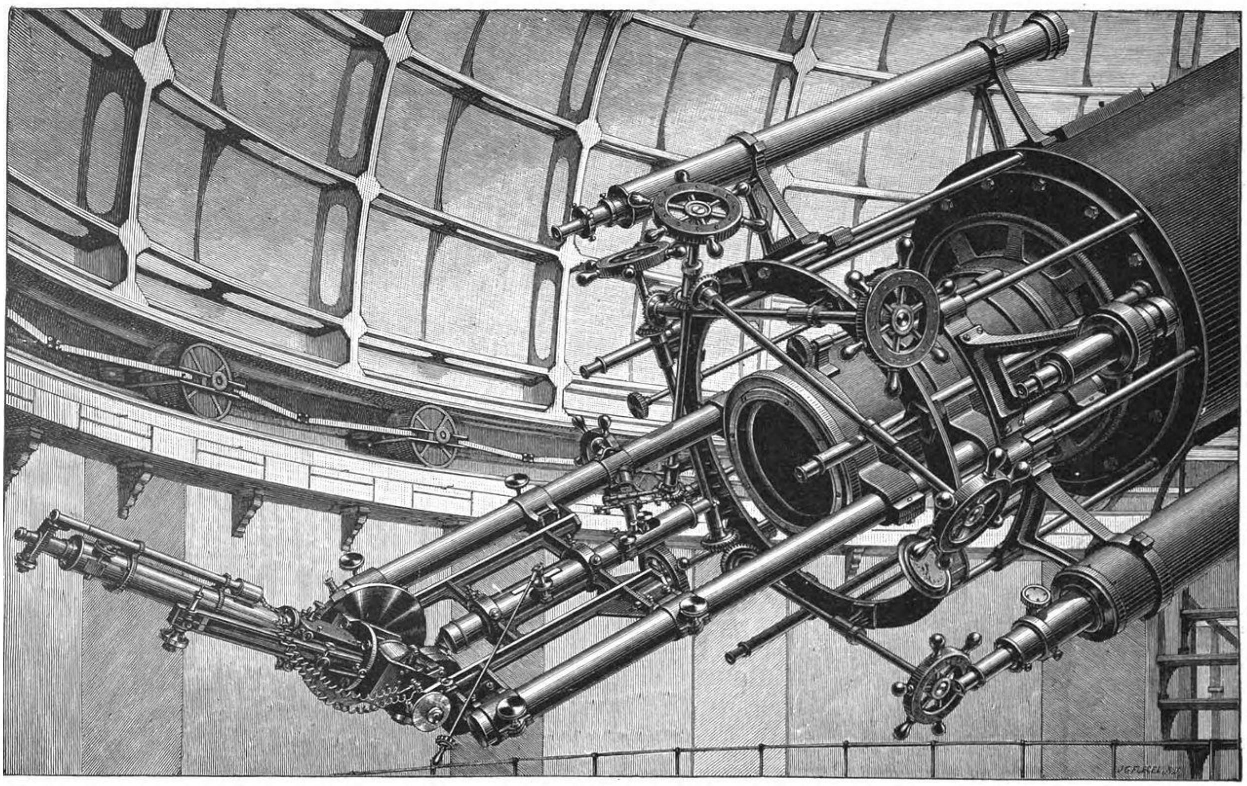


Figure 30 - The Lick Observatory Star Spectroscope (Credit 1898 book "*Treatise on Astronomical Spectroscopy: Being a Translation of Die Spectralanalysis Der Gestirne*" by Julius Scheiner.)

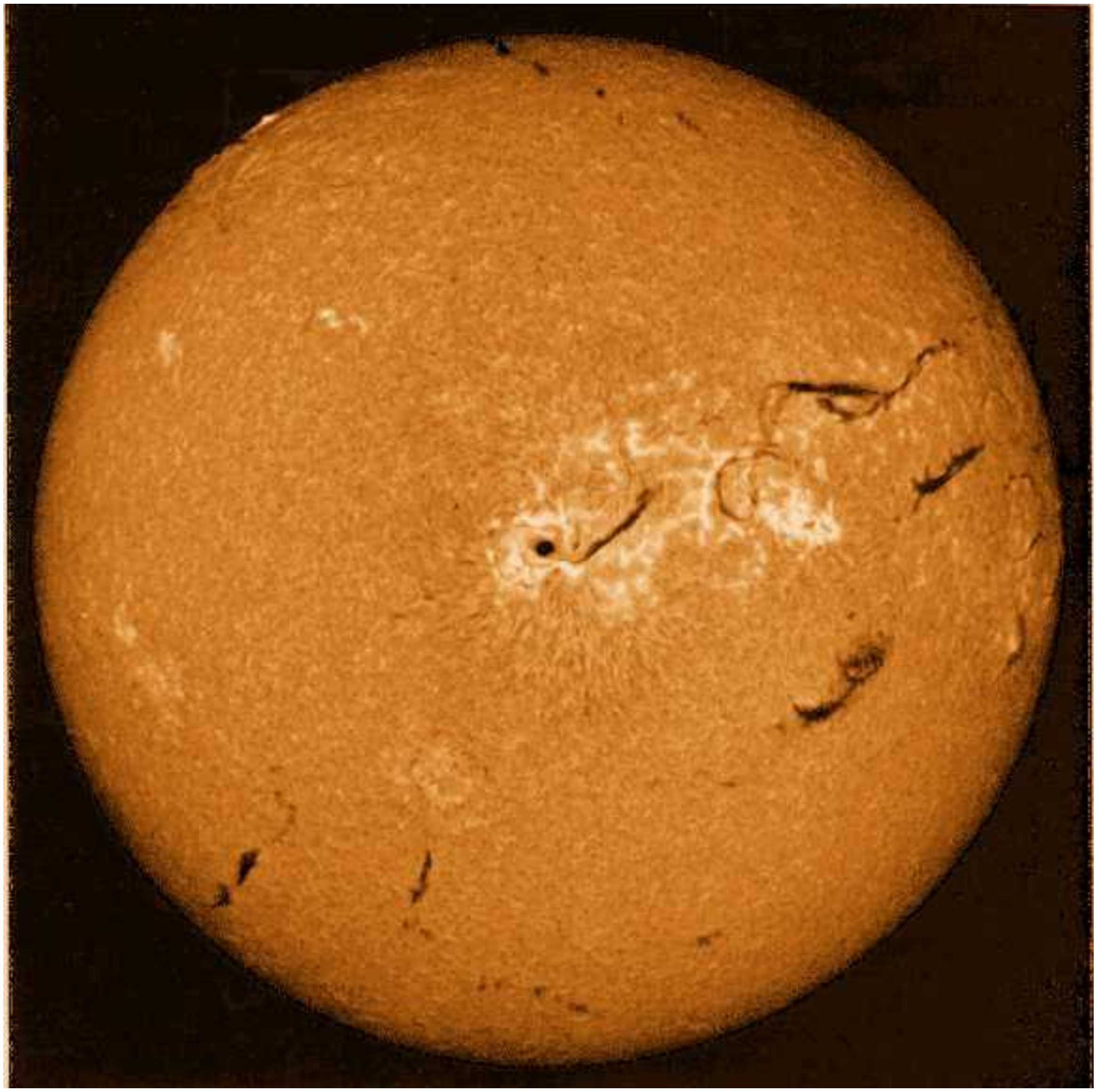


Figure 31 - This is an image of the Sun as seen in H-alpha. H-alpha is a narrow wavelength of red light emitted and absorbed by the element hydrogen at a wavelength of 656.28 nm (0.65628 μm). *(Courtesy National Solar Observatory/Sacramento Peak)*

The simultaneous measurement of the light from hundreds of galaxies at a time can now be accomplished with the help of fiber optics. In a multi-fiber spectrometer, a metal plate fits on the focal plane of the telescope and is drilled with hundreds of holes. Each hole is centered on a particular galaxy to be imaged at a particular sky location. Into each hole, a single fiber optic cable is attached. The light from the galaxy is carried to the spectrometer and becomes its own spectrum of light. Over one hundred galaxies each hour can be analyzed in this way and their doppler shifts determined by a computer program.

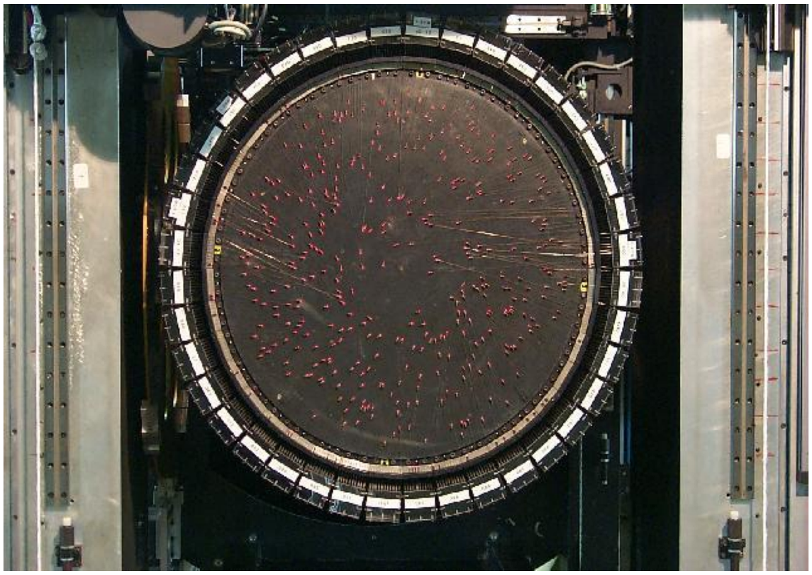


Figure 32 – The focal plane plate from the Hydra instrument at the Australian Astronomical Observatory (Credit: SAMI Instrument)

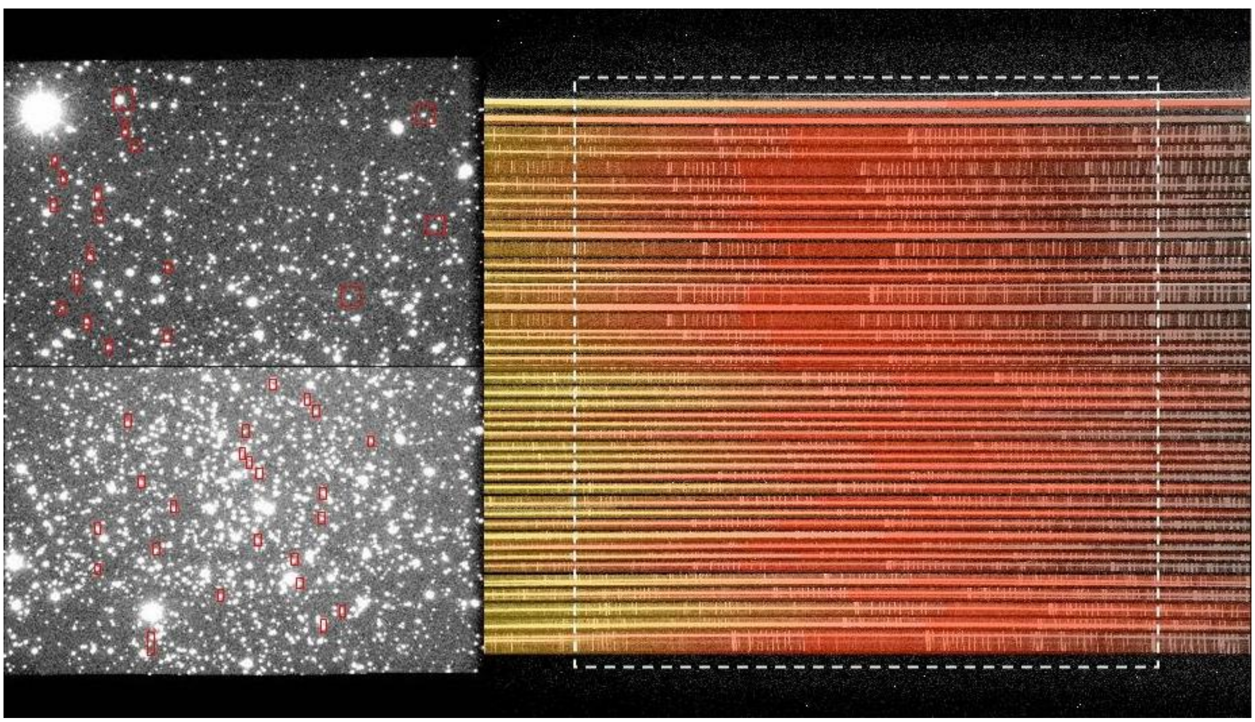


Figure 33 - The image on the left is a globular cluster with red rectangles marking the location of the entrance slits for the aperture mask. The spectra on the right shows the output from the multi-fiber spectrometer. (Credit: Keck Observatory LRS)

Spectroscopy is a technique that is use at all wavelengths of the EM spectrum because it allows astronomers to dissect the light and search for the fingerprints, called spectral lines, of a variety of elements and molecules. Atomic lines are commonly found at wavelengths corresponding to the visible spectrum below 1-micron. Some spectral lines from simple molecules such as titanium oxide (TiO) can also be found at visible wavelengths. For more complex molecules such as water, carbon dioxide and ammonia, the spectral lines are found in the infrared region at wavelengths longer than about 1-micron, which is why astronomers want to explore this part of the EM spectrum.

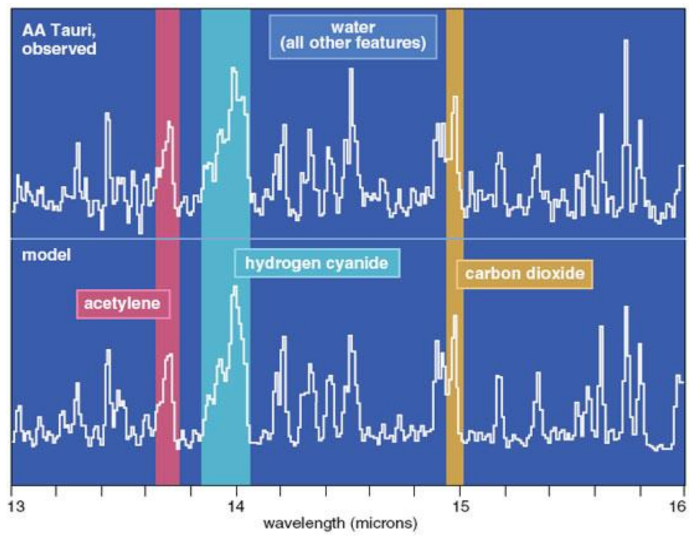


Figure 34 – Example of spectral lines in the infrared. Adapted from image courtesy of NASA/JPL-Caltech/J. Carr [Naval Research Laboratory].

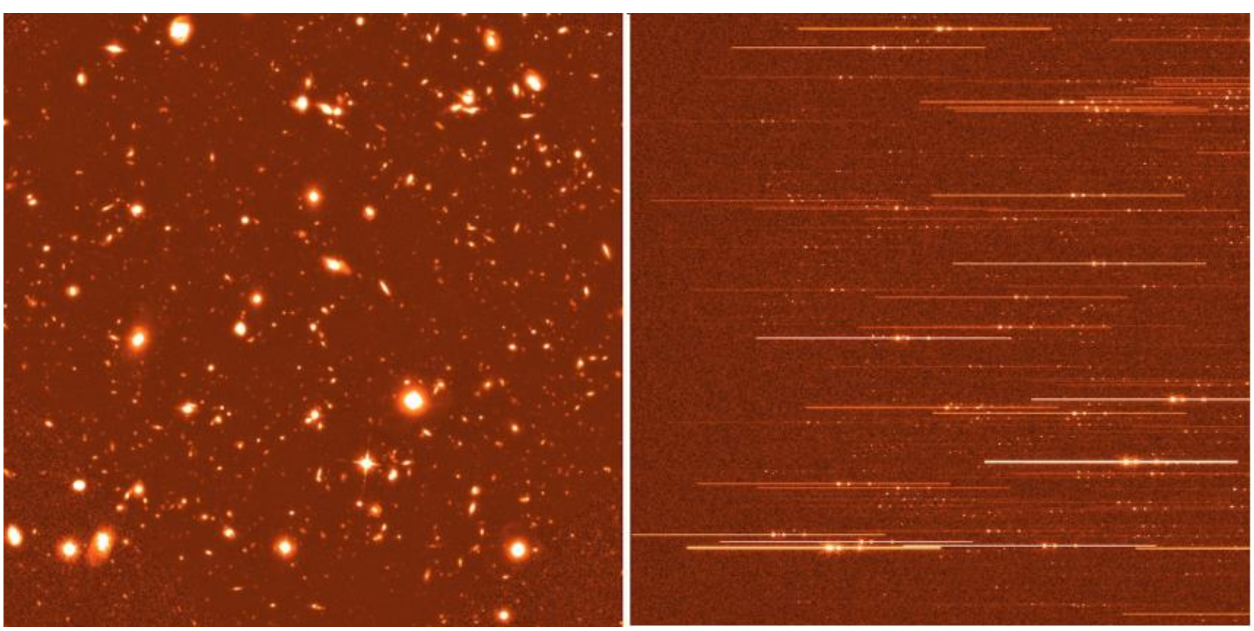


Figure 35 - Left: Image of the Hubble eXtreme Deep Field covering the area around the Hubble Ultra-Deep Field; Right: Simulated 2-hour Webb NIRCam slitless spectrum (wavelength increases toward left). (Credit: NASA)

Figure 36 - A photo of the Milky Way with a smartphone. (Credit: Gabe Clark).

IV – Introducing the James Webb Space Telescope - JWST

Goal: Students learn about the basic operation of the Webb Space Telescope and how it 'takes pictures' in the infrared spectrum.

From all of what we have covered so far, the infrared part of the EM spectrum has proven to be a scientific bonanza for studying many important classes of astronomical objects from remote, cold asteroids in our solar system to the most distant and ancient stars and galaxies in the visible universe. Since the launch of the IRAS satellite in 1983, many subsequent infrared telescopes have steadily improved our understanding of the universe each time the size of the aperture doubled in size. The most interesting and versatile region of the EM spectrum is just beyond what the Hubble Space Telescope could observe and extends from 0.8 to 30-microns covering the near and mid-infrared regions.

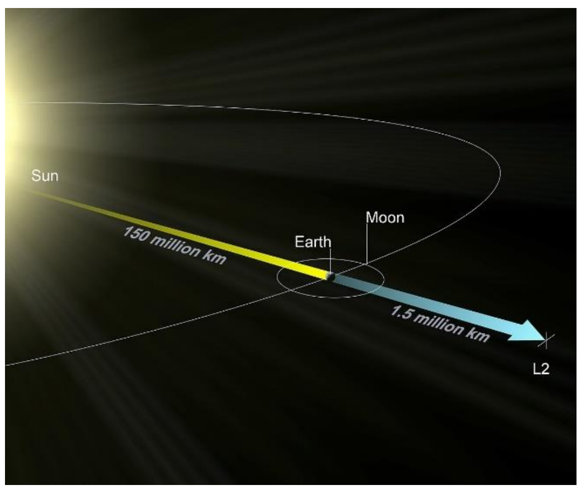


Figure 37 - The JWST will be placed at the L2 'Halo Orbit' and maintain its position there by using its station-keeping thrusters occasionally. This propellant gas is what limits its lifespan to not more than 10 years. (Credit: NASA)

Following the success of NASA's Spitzer Space Telescope (2003-2020) a companion to this telescope but with a far-larger mirror was developed starting in 1996 and construction began in 2003. While Spitzer and previous telescopes orbited Earth, which is a warm body whose heating of the telescope would be hard to control, the plan was to place JWST parked at the L2 Lagrange Point located about 1.5 million km (930,000 miles) from Earth. From this location, there would be no other significant heating to the telescope except for the sun, a sunshade could keep the telescope in a state of permanent shade.

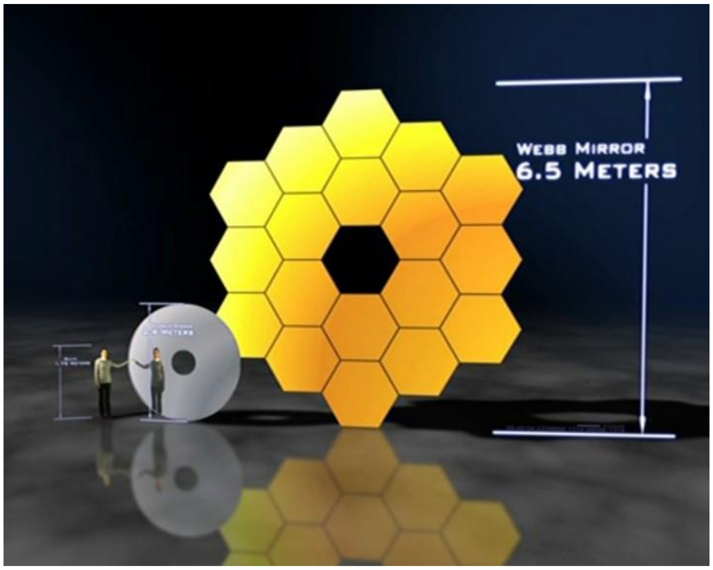


Figure 38 - A comparison of the hexagonally-tiled JWST primary mirror with the mirror used by the Hubble Space Telescope. (Credit NASA/JWST)

The problem at the start of the design phase was how to fit a nearly 7-meter mirror and the sunshade into the small-diameter shroud of the Ariana rocket. Engineers soon decided that the mirror and the sunshade would have to unfold like a flower petal, which was a deployment method that had never been tried before.

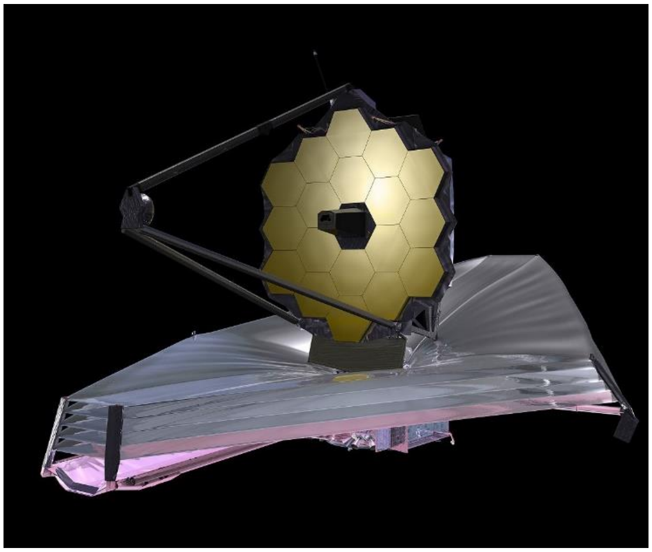


Figure 39 - The JWST rendered in a simulated model showing its basic components. (Credit: NASA/JWST)

Many simulations of the design were created until after years of work the current design for the telescope was locked-in in 2007 and the way was opened for the actual fabrication to begin. The fabrication and testing phase took longer than anticipated, but finally the $10 billion telescope payload was 'buttoned up' and readied for launch in 2021. Including the United States, 20 counties have contributed instruments and components to this telescope.

Problem 12 - The Launch of the Mars Science Laboratory (MSL) in 2011

Problem 13 - Scaling Up the Webb Space Telescope Mirror

Problem 14 - The Hexagonal Tiles in the Webb Space Telescope Mirror

Problem 15 - 6-fold Symmetry and the Webb Space Telescope Mirror

Problem 16 – Gold film on the JWST primary mirror.

Table 5 – Basic features of the Webb Space Telescope

Item	Value
Launch date	October 31, 2021
Launch vehicle	Ariane 5 ECA
Mission duration	5-10 years
Payload mass	6,000 kg
Electrical Power	2,000 watts
Primary mirror	6.5 meters (21.3 feet)
Area of mirror not obstructed	25 m^2
Primary mirror composition	Beryllium coated with gold
Mass of primary mirror	705 kilograms
Mass of one of 18 single hexagonal segments	20.1 kg for mirror tile and 19.4 kg for mount
Mirror focal length	131.4 meters
Optical resolution	0.1 arcseconds
Wavelength coverage	0.6 to 28.5 microns
Size of sunshield	21.2 x 14.2 meters (69.5 x 46.5 feet)
Temperature range of sunshield	383 to 221 kelvins
Temperature of telescope	39 Kelvins

A – Providing power to the telescope from sunlight

You may notice from the many pictures of the telescope that there do not seem to be any obvious solar panels to produce the 2,000 watts of power for the telescope. That's because the solar panels are on the less-photogenic, under-side of the sunshades and are attached to the spacecraft bus.

Figure 40 – Bottom view of the JWST showing its solar panels. (Credit: NASA/JWST)

The telescope requires 1,000 watts of power but the solar array is able to produce 2,000 watts so that the telescope can still operate for many years as the solar panels degrade.

Figure 41 – Technologists holding the solar array of the James Webb Space Telescope after a test of the array earlier in 2020. The fifth panel of the solar array is folded behind the fourth in this image. (Credit: NASA/Chris Gunn)

Problem 17 - Hinode Satellite Power

B – Telemetry

How do satellites and spacecraft get their precious cargo of data back to Earth? The answer is 'remote measuring' or telemetry. In robotics, you have a radio transmitter that is human-operated, controlling the movement of a robot without physical contact. Spacecraft take measurements with their various sensors and use an onboard transmitter to relay the data back to Earth to get recorded and later processed into images and research papers. In robotics, the radio link is accomplished using a transmitter-receiver set at a fixed frequency that can be used by many robotic systems. With remote spacecraft, the frequency has to be carefully matched to the rate at which the data is being taken by the instruments. The reason for this is that for a given frequency, there is a maximum amount of data that can be transmitted in the most elementary form of computer binary 1s and 0s. This frequency is called the Nyquist frequency.

A radio wave is a continuous sine-wave that has alternating positive and negative parts; one pair per cycle. A binary stream of data needed to transmit images consists of 1s and 0s. For a radio wave to transmit 1s and 0s accurately, you need one cycle for each bit of data (1 and 0) in the data stream. If the data rate is C=1 bits per second, you need a radio wave that has a bandwidth of 2B or 2 cycles-per-second to accurately record this binary datum. If you use a radio wave that has more than this frequency, it is just making multiple, redundant (wasteful) measurements, so the Nyquist frequency is the optimal frequency to transmit the data without duplications of measuring a given bite of information. In fact, there are two parts to a radio signal. The first part is the frequency of the carrier signal. The second part is the frequency width over which the carrier is being transmitted, also called the bandwidth. The Nyquist frequency determines the bandwidth, but the carrier frequency has to be high enough to support the bandwidth. For example, if the Nyquist bandwidth required is 1,000 Hertz, you can accommodate this on a telemetry signal at a carrier frequency of 5,000 Hertz but not is the carrier frequency is lower than 1,000 Hertz. So, for satellite data, how do we figure out the maximum data transmission rates and carrier frequency?

Suppose you have a digital camera that has 1000 pixels and each pixel brightness value has 8 levels. You want to transmit the image data after each exposure back to Earth, and you plan to take one exposure every five seconds. You are going to take 10 images and store them in a computer memory area and then transmit the contents of this 'buffer' back to Earth in a telemetry stream. This is a typical situation that an engineer faces in designing a satellite data telemetry system. The way to work out its details is as follows:

First, each pixel requires 3-bits to record its 8 possible values ($2^3 = 8$), so a single image requires 3 bits/pixel x 1000 pixels = 3,000 bits of data.

Next, you will store 10 of these images in the buffer, so you need a memory capacity of 10 images x 3,000 bits/image = 30,000 bits. In terms of bytes where 1 byte = 8 bits, this memory capacity equals 30,000/8 = 3,750 bytes. Your smartphone, by comparison, can store 32 gigabytes of data before it 'overflows'. Buffer overflow is a hazard for data-gathering because you will lose valuable data if this happens and there is nowhere for the system to store the new data.

Now that we have sized the buffer to 3,750 kilobytes, we have to transmit this data and clear the buffer in time for the next 10 images to start being stored. It takes one second for an image to be taken so we have one second to download the data before the 11th image has to be written into the buffer. So, we need a data rate of 30,000 bits per second. From the Nyquist frequency, data rate is equal to twice the frequency (c = 2f) so we need a frequency of 15,000 Hertz as the signal bandwidth. That means the carrier frequency needs to be at least 15,000 Hertz.

So, to transmit to Earth the image data and buffer it properly, we need a buffer with a capacity of 3,750 kilobytes, a bandwidth of 15,000 Hertz, and a carrier frequency of at least 15,000 Hertz. Let's see how these calculations work for the Webb NIRcam instrument.

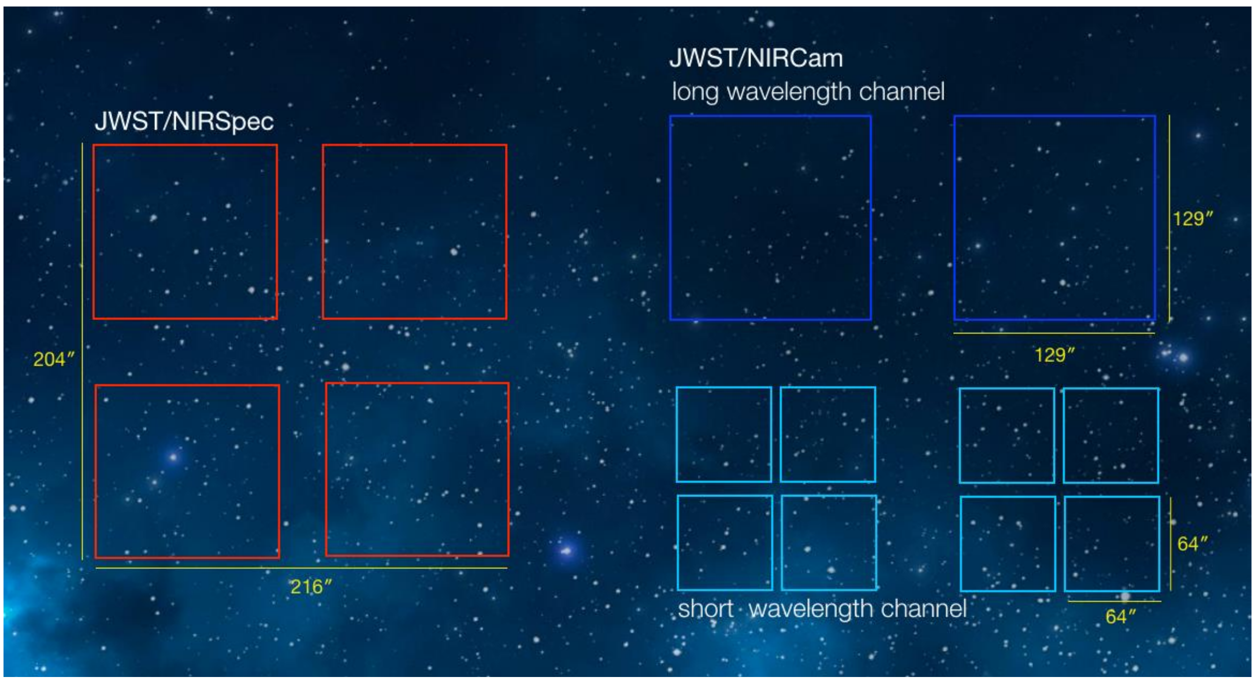


Figure 42 – Here is what the fields look like for the NIRcam instrument (right) and the NIRSpec imaging spectrometer (left). Although the long-wave (LWC) camera and the short-wave (SWC) camera look at the same sky location, the figure shows them offset so that you can see their sizes more easily. (Credit: NASA/JWST).

The goal of the NIRcam is to take diffraction-limited images of astronomical objects. The imager consists of an array with 2048 x 2048 pixels, and one image is 8 megabytes. The field-of-view of the entire instrument has a footprint that spans an extent of 5.1' × 2.2' on the sky. Each pixel samples an angle of about 0.032 arcseconds in the short-wavelength band (SWC) and 0.65 arcseconds in the long-wavelength band (LWC). In Figure 43 you see how the resolving power of

JWST makes a big difference in how clearly you can see objects compared to the smaller Spitzer Space Telescope.

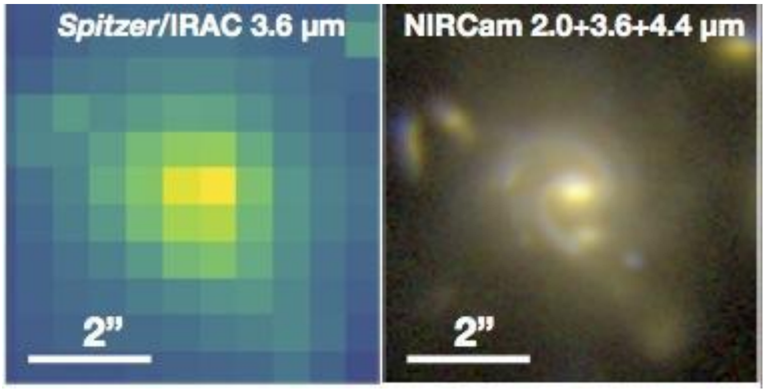


Figure 43 - The James Webb Space Telescope will give scientists a new level of detail in studies of the structure of galaxies. This side-by-side image compares the resolution of a distant galaxy imaged by the Spitzer Space Telescope in the near-infrared, left, to a simulated NIRcam image from JWST. (Credit: NASA, CEERS team)

Downlinks for recorded science data occur in 4-hr contacts twice a day, with each contact transmitting at least 28.6 gigabytes. Onboard storage is provided by a solid-state recorder with a capacity of 58.9 gigabytes. Operating like a digital video recorder, the spacecraft flight unit records all science data together with continuous engineering "state of health" telemetry for the entire observatory 24 hours a day, seven days a week. The data is downloaded to the ground station when the telescope communicates with Earth during a four-hour window every 12 hours. The Ka-band downlink operates at a carrier frequency between 27 to 40 gigahertz with a 10-megahertz bandwidth limit, and is used for science data at the selectable rates up to 28 Mbit/s. The Ka-band transmitter uses a 0.8-meter dish located on the underside of the telescope, and transmits at a power level of about 25 watts. The NASA Deep Space Network, with its large 34-meter dishes at Canberra, Australia; Madrid, Spain; and Goldstone, California, will receive the Ka-band data and then relay it via microwave links to the science institutions for processing and analysis.

Table 6 – Telemetry rates for NASA DSN data types

Readout pattern	Time between groups	Data Volume (GB/day)	Data Volume (MB/s)
*DEEP2, DEEP8**	~200s	~34	0.39
MEDIUM2, MEDIUM8	~100s	~68	0.79
SHALLOW2, SHALLOW4	~50s	~136	1.6
BRIGHT1, BRIGHT2	~20s	~340	3.9
RAPID	~10s	~680	7.8

Problem 18 - Digital Camera Math

Problem 19 - Exploring the InSight Lander Telemetry Data Flow

Problem 20 - Advanced Unit Conversions

V -What to expect from JWST images

Goal: Students explore what Webb Space Telescope data may look like and how to interpret it using data from the Spitzer Space Telescope.

As with all images and data from a new telescope, there is enormous excitement to see what the new technology will bring. Every telescope has its own character in the tempo and formats that the data presents. Long before a spacecraft arrives at its destination and opens its cover for 'first light' the telescope's observing time has been scheduled to the second. For imaging instruments, the telescope is pointed at both familiar and unfamiliar objects to gather data at higher resolution or at wavelengths previously unexplored.

The JWST, with its huge mirror, promises views of its target objects and star fields that will be deeper than anything previously studied. As a companion to the Hubble Space Telescope, it operates beyond the red limit of the Hubble Space Telescope and allows astronomers to compare Hubble's optical images with images deeper into the infrared. We can get a good idea of what to expect from JWST by taking a look at the data provided by the Spitzer Space Telescope launched in 2003.

A-The Spitzer Space Telescope

Goal: Students learn basic information about the Spitzer Space Telescope and its research

The Spitzer Space Telescope debuted in 2003 and obtained its final images in 2020 once its liquid helium coolant ran out. Spitzer was one of NASA's four Great Observatories, along with the Hubble Space Telescope, the Chandra X-ray Observatory and the Compton Gamma Ray Observatory. The Great Observatories program demonstrated the power of using different wavelengths of light to create a fuller picture of the universe. Its infrared images obtained between wavelengths of 3 to 160-microns were often compared side-by-side with Hubble images taken at optical wavelengths from 0.1 to 2-microns.

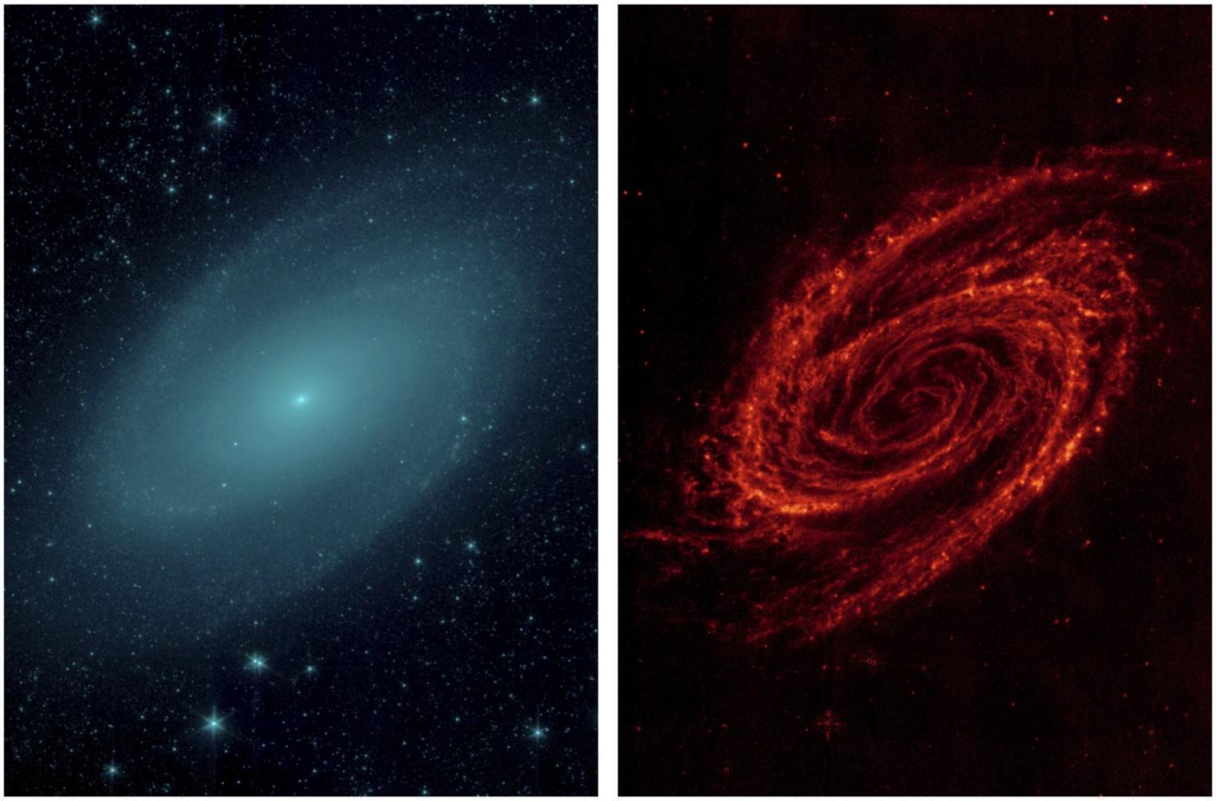


Figure 44 - This combination of photos made available by NASA shows the spiral galaxy Messier 81 (M81) viewed in two different types of infrared wavelengths showing the light from the stars in the galaxy at 3.6-microns, left, and the light from only the dust particles at 24-microns, right. The dust particles are similar to beach sand but are ten times smaller. (NASA/JPL-Caltech via AP)

As Figure 44 shows, a galaxy typically looks very different in the infrared than at optical wavelengths. The reason for this is that stars are very powerful sources of optical radiation and completely dominate the visible shape of a galaxy. But when you look at some galaxies like our Milky Way, you can see its arms are often crisscrossed by obscuring dust clouds that absorb the starlight. These dust clouds are of keen interest to astronomers because it is deep inside them that new stars are being born. These stars heat up their surroundings and so cause some portions of these dust clouds to glow in the infrared. What infrared telescopes such as Spitzer were able to do is to strip-away the light from the stars and show images of galaxies defined only by the shapes and locations of these infrared clouds. Figure 44 shows one such galaxy, Messier-81, which shows the galaxy in terms of its starlight at 3.6-microns, but in the right-hand image at 24-microns you only see the glow of the dust clouds. These clouds are lumpy in texture because they are filled with young, luminous star-forming regions. What is also exciting is that these clouds follow a spiral pattern just like the optical view but the clouds are shifted towards the inner edges of these spiral arms, revealing new clues about how star formation is triggered.

Although some galaxies are so distant their shapes cannot be readily discerned, nevertheless, a different kind of instrument can reveal new information about them. When the light is put through a specially-designed infrared spectroscope, the fingerprint lines from different important molecules can be revealed. But consider this; instead of looking at the light from a single star or dust cloud, we are taking a survey of what all the material in the galaxy is doing, along with its composition. Figure 45 is the data from a spectroscopic study of the very luminous infrared galaxy called IRAS F00183-7111 located 3.2 billion light years from the sun. This galaxy is also called a quasar, and its enormous light output is, like so many other quasars, a bit of a mystery. Astronomers would like to gather as many clues about quasars and luminous infrared galaxies so that they can figure out how the light is produced. For this quasar, Spitzer was able to detect the fingerprints of many important molecules needed for living things including water, carbon dioxide, hydrogen and other hydrocarbon compounds.

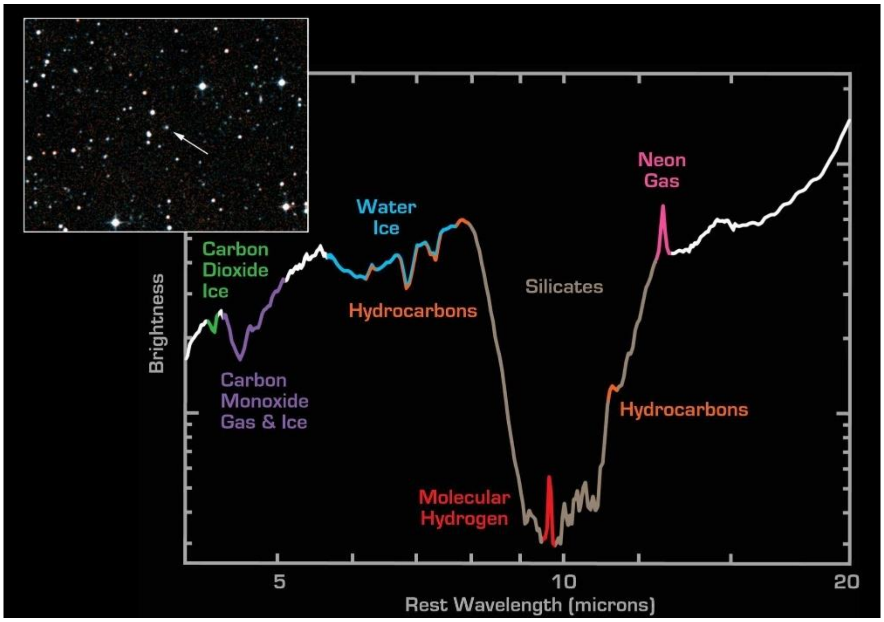


Figure 45 - NASA's Spitzer Space Telescope has detected the building blocks of life in the distant universe, albeit in a violent milieu in the galaxy IRAS F00183-7111. The infrared spectrograph instrument onboard Spitzer breaks light into its constituent colors, much as a prism does for visible light. (Credit: NASA /JPL /Caltech)

In Figure 45, the magenta peak corresponds to singly ionized neon gas, a spectral line often used by astronomers as a diagnostic of star formation rates in distant galaxies. The spectrum obtained by Spitzer is the result of only 14 minutes of integration time, highlighting the power of the infrared spectrograph to unlock the secrets of distant galaxies.

Closer to home, the detection of thousands of exoplanets orbiting other stars has created a surge of interest in finding Earth-like planets in size, mass and temperature that might possibly be abodes for life. Although we will never be able to see actual organisms from this distance, we can still look for the ways in which biospheres alter their atmospheres through respiration. On

Earth, the first 2 billion years had bacteria that respired oxygen and so this molecule built up in the atmosphere. Then about 2 billion years ago, more advanced bacteria switched over to using oxygen as an energy source. Meanwhile, vulcanism continued to emit nitrogen, water vapor and carbon dioxide. The nitrogen built-up so that today our atmosphere is about 70% nitrogen and 22% oxygen with trace gases of carbon dioxide, argon and other molecules. One of the goals of Spitzer was to study a few of these exoplanets and detect their atmospheric constituents. At the present time about a dozen exoplanets have been studied in this way with Spitzer before its mission came to an end.

Figure 46 shows a combined Hubble and Spitzer study of the exoplanet WASP-39b. This exoplanet is classified as a Hot Jupiter with a mass of about 25% of Jupiter, but its 4-day orbit would be entirely inside the orbit of our planet Mercury. It has a dense atmosphere, which is very hot at a temperature of 1,430° F (777 C). The main atmospheric constituents seem to be water vapor, and carbon dioxide.

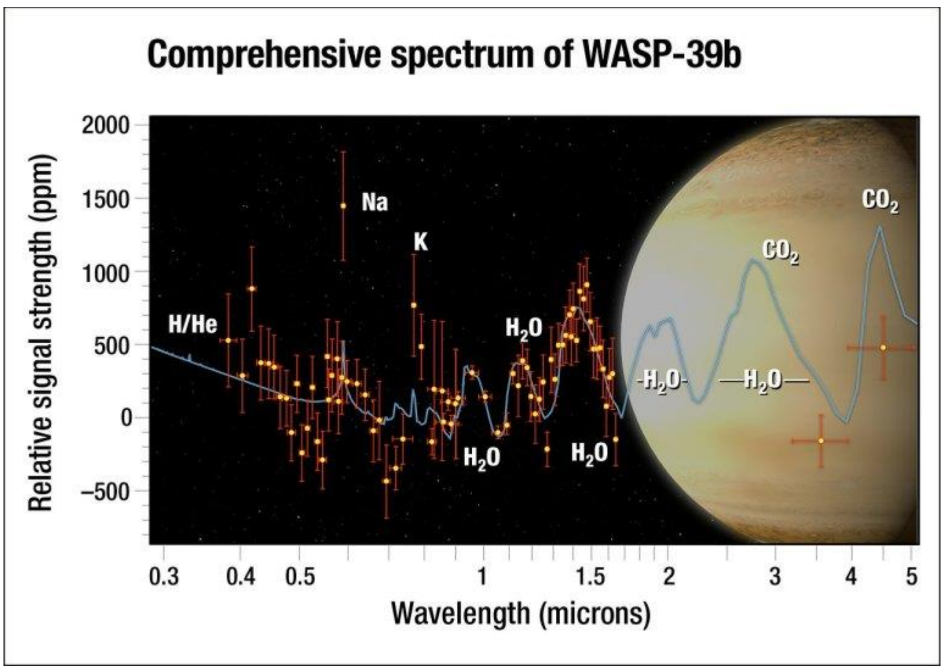


Figure 46 – Infrared spectrum of WASP-39b created by combining the data from Hubble (0.3 to 2-microns) and Spitzer (2 to 5-microns). (Credit: NASA/Spitzer)

Problem 21 - The Most Important Equation in Astronomy

Problem 22 - Spitzer Telescope Discovers New Ring of Saturn!

Problem 23 - Hubble Sees a Distant Planet

B – The Webb Space Telescope

Goal: Students examine what astronomers expect to see with the Webb Space Telescope

Exploring the edge of time

Because it takes light time to get from place to place, when we look at things anywhere in the universe, we are always seeing them as they were long ago. This is a huge advantage of we want to investigate what the universe and the objects in it, looked like and how they formed when they were very young. Our universe is 13.8 billion years old, but our ground and space-based instruments can take images of infant galaxies that formed only a few hundred million years after the Big Bang.

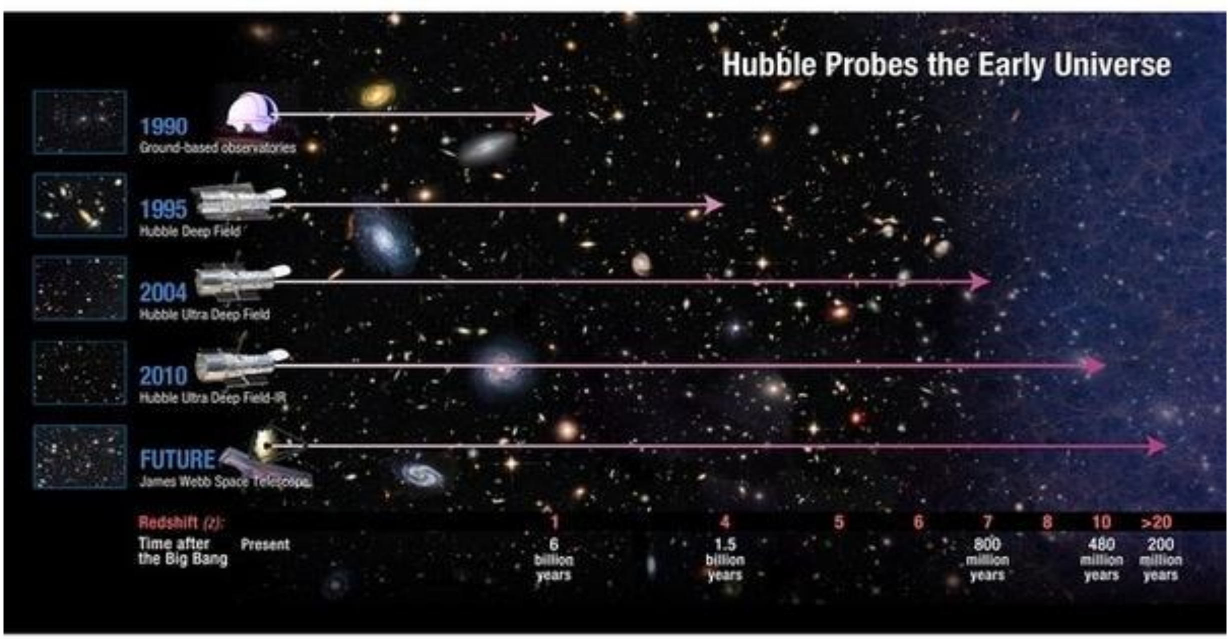


Figure 47 – The JWST will be able to look much deeper into space and therefor much farther back in time that previous telescopes. (Credit NASA, ESA and A. Field: STScI)

One of the features of our universe is that it is expanding, but that it is space-itself that is stretching and not that objects are moving rapidly through it. What this also means is that, when a visible light ray at a wavelength of 0.5-microns was first emitted by a remote object, its wavelength has stretched enormously. By the time this light ray reaches our instrument after traveling 13 billion years, its wavelength can be stretched by the expansion of space to more than 10-microns. This means that if you want to study what a galaxy looked like in the infant universe, although it may have been a blazing clump of stars emitting visible light, your instrument will record its image as a dull red infrared source. The Hubble Space Telescope could study galaxies out to 10 billion light years, but for more distant objects, their light was 'red shifted' beyond the portion of the spectrum Hubble could study. Not only will JWST go beyond Hubble and detect galaxies to 13 billion light years, but its larger mirror will be able to collect more of their faint light. Figure 48 and 49 are examples of what such views might look like compared to what Hubble is able to do. Not only are there far more faint galaxies to be found, but with the larger JWST mirror, the images are much clearer and better-resolved.

Problem 24 – Working with expanding space

Problem 25 – How big is the universe?

Problem 26 – Distance and look-back time.

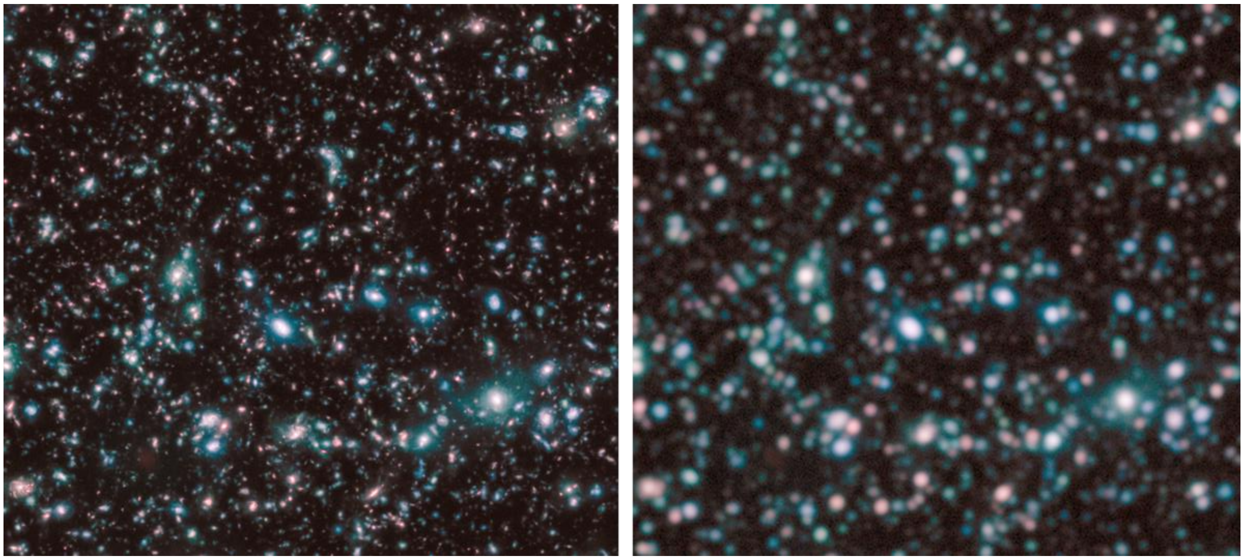


Figure 48 - Shown here are simulated Spitzer (top) and JWST (bottom) images of distant galaxies in infrared colors. These were constructed from a computer simulation of the deep universe. (Credit: G. Snyder & Z. Levay - STScI)

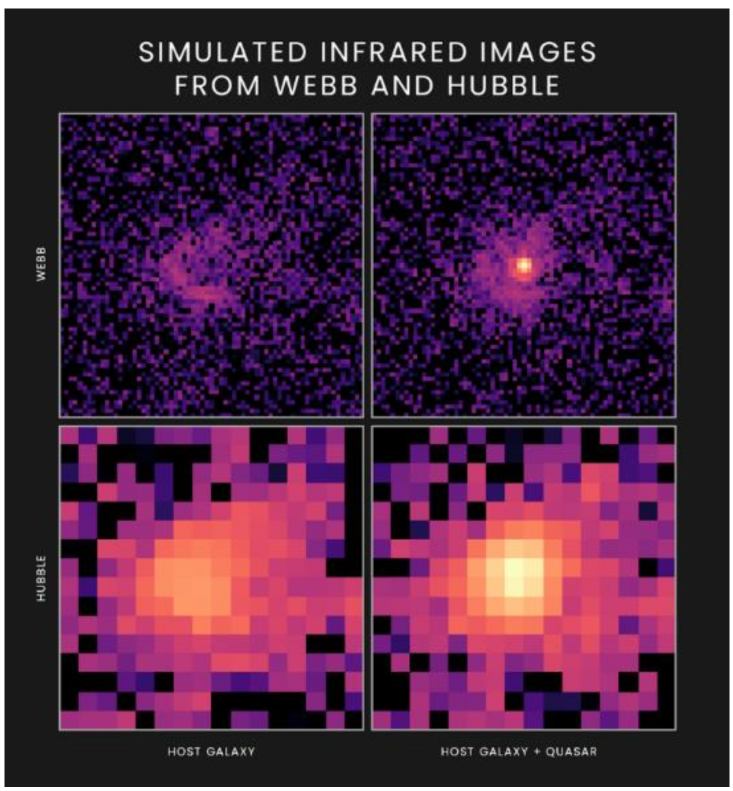


Figure 49 - The simulated images below show how a quasar and its host galaxy would appear to NASA's upcoming James Webb Space Telescope (top) and Hubble Space Telescope (bottom) at infrared wavelengths of 1.5 and 1.6-microns, respectively. (Credit: M. Marshall, University of Melbourne).

With infrared spectroscopy, JWST comes into its own by being able to study the fingerprint lines from a host of important molecules. Some of these, such as silicates, indicate the presence of rocky dust grains formed in the atmospheres of red giant stars, but now incorporated into interstellar dust clouds. Other molecules such as water, methane and oxygen are important for living systems similar to Earth life. Figure 50 shows one such simulated spectrum of a dust cloud in the Eagle nebula and how JWST may reveal its molecular constituents.

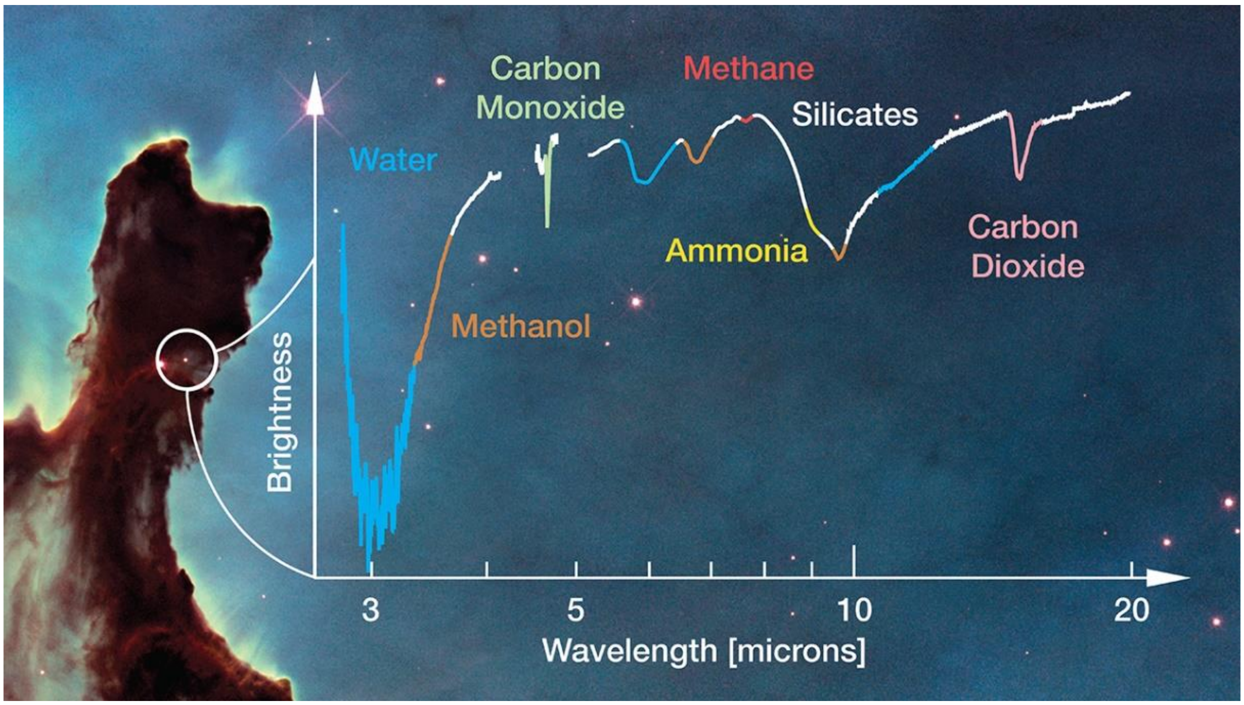


Figure 50 - This simulated spectrum from the Webb telescope illustrates the kinds of molecules that may be detected in star-forming regions like the Eagle Nebula (background). (Credit: NASA, ESA, the Hubble Heritage Team (STScI), M. McClure (Universiteit van Amsterdam) and A. Boogert (University of Hawaii))

Closer to home, JWST will let astronomers study some of the exotic chemical sites in our own solar system. Figure 51 shows a view of what the Jovian satellite Europa would look like when its NIRcam pixels cover its disk. At this resolution, one can resolve features only about 300 km across (one pixel) but the infrared spectrometer NIRSpec can detect complex organic molecules in the plumes of water that this ide-covered world ejects. This allows astronomers to sample the inaccessible liquid ocean below the surface of Europa and check whether it is an organic soup of chemicals out of which life may have emerged. Because Europa is nearby, it can also be observed multiple times every year to see how its environment changes from the geyser activity.

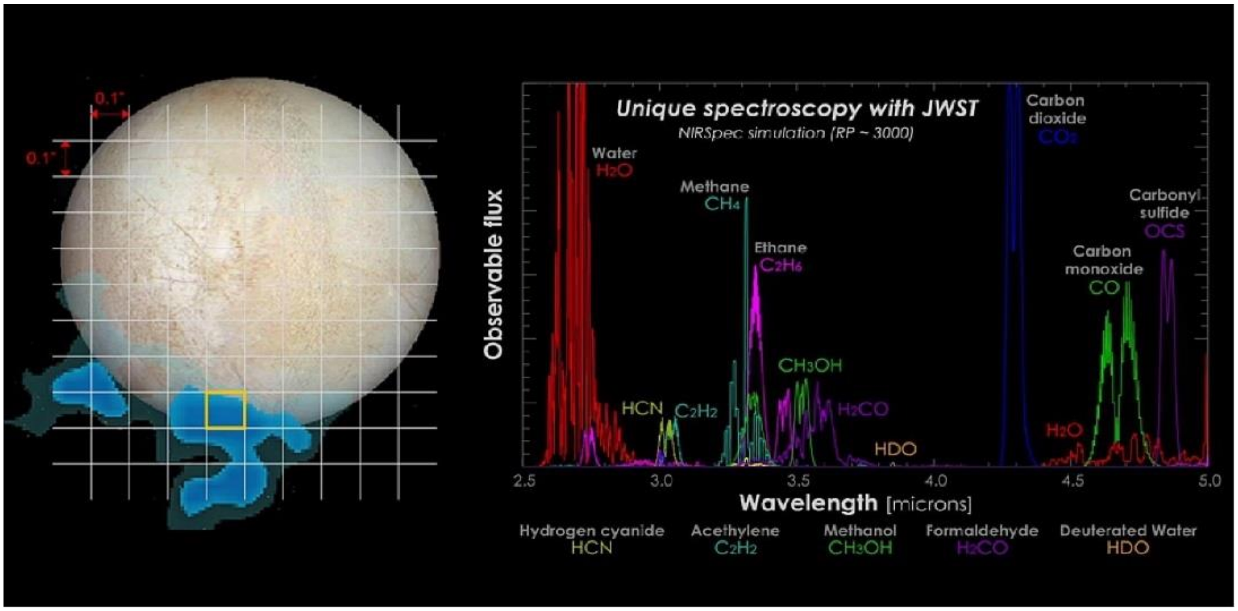


Figure 51 - Possible spectroscopy results from one of Europa's water plumes. This is an example of the data the Webb telescope could return (Credit: NASA-GSFC/SVS, Hubble Space Telescope, Stefanie Milam, Geronimo Villanueva)

Another intriguing locality in the solar system is Mars. Its atmosphere was once thought to be largely pure carbon dioxide, but in the last 20 years traces of more complex molecules have been detected. The search for signs that life once flourished on this desiccated world continues with an armada of orbiting spacecraft and surface rovers such as Curiosity. The JWST can carry these investigations to another level by searching for complex organic molecules in the Martian atmosphere. One of these, methane, has been detected by Earth-based telescopes but does not remain as a fixed concentration. Instead, it rises and falls with the Martian seasons in the equatorial zone and provides an intriguing clue that some organic process (life?) may be producing it via respiration. JWST will be able to keep watch on Mars periodically and determine what other molecules may also be present.

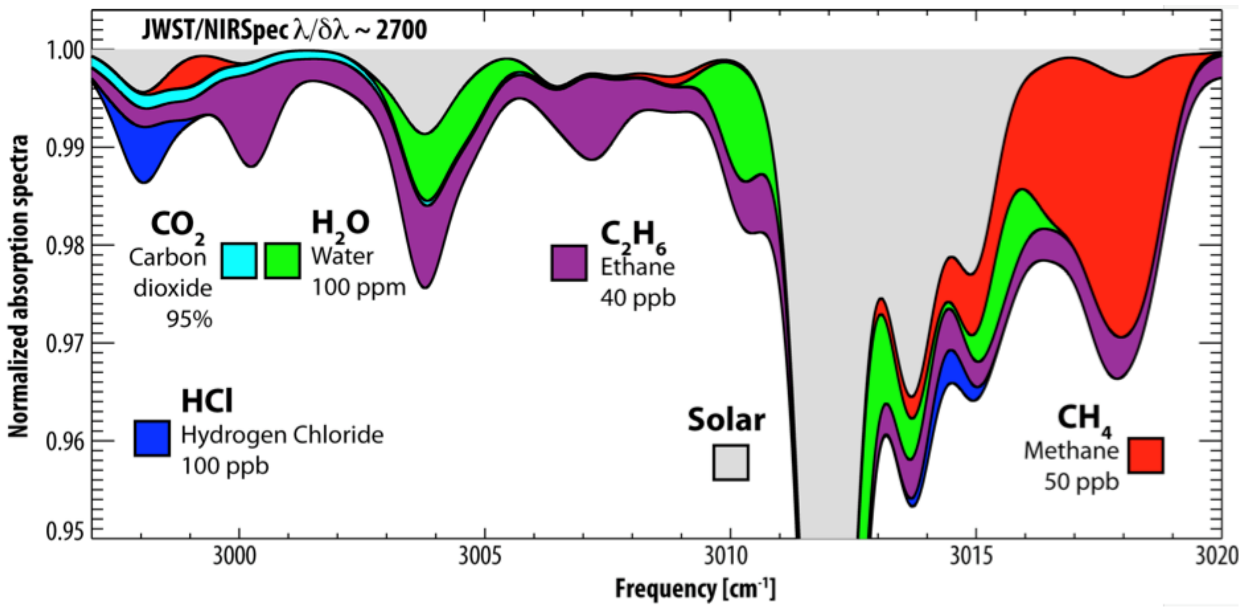


Figure 52 – With JWST, searches can be conducted for organic molecules in the Martian atmosphere. (Credit: G. Villanueva, NASA /GSFC).

Problem 27 - Seeing the Distant Universe

Problem 28 - Webb Space Telescope: Detecting dwarf planets

Problem 29 - The Cosmological Redshift - Changing the light from a galaxy

VI-Hands-on experiments.

The NASA, Science, Technology, Engineering, Arts and Math (STEAM) Lab was a makerspace approach to bringing NASA's universe of knowledge into educational spaces around the country and to increase scientific literacy and interest in STEAM-related careers. Through a series of stations specializing in virtual reality, electronics, mobile sensors and 3-D fabrication, the NASA team succeeded in impacting over 82,000 educators and students, along with 1.6 million social media participants. It was inaugurated at the NASA Goddard Space Flight Center in Greenbelt, Maryland in 2012. It was a one-room resource center and website offering the teacher, scientist, or engineer a variety of tools in areas such as Virtual Reality, 3-D printing, and Mobile Sensors, with the aim of fostering both education and DIY opportunities for creative 'play'. Its goal was to be a catalyst for generating ideas using technology and to stimulate education for teachers and learners alike. An individual or a group could propose a project idea and submit it to the SIL community of partners. The feedback provided by the broader NASA Space Science Education Consortium (NSSEC) community through the STEAM Innovation Design Experiment telecons could then be used by the developer to adjust and improve their design, fabrication or dissemination approaches, and identify other partners interested in helping out.

A defining feature of SIL that sets it apart from a typical makerspace was that it took advantage of NASA’s scientific and engineering knowledge base to infuse prospective projects with science and engineering content. The following experiments were created to highlight the infrared principles that make the JWST possible through a variety of innovating and low-cost experiments.

A-Exploring black body curves

The manner in which the light from a heated object varies with wavelength is not hap-hazard but follows a precise curve called the black body curve. It gets its name from the face that a surface that perfectly absorbs all radiation that falls on it also emits radiation when heated. This radiation follows exactly the black body curve.

Required Material:

- ✓ Laptop and access to internet.

Procedure:

Step 1) Visit the website curve calculator at https://www.spectraplot.com/blackbody

Step 2) On the top of the page, enter 300 in the box for T(K) to select a temperature of 300 Kelvins

Step 3) In the windows on the second column, in the top box enter 5 and the bottom enter 50 to select a plot range from 5 to 50-microns.

Step 4) Click the bar to the right that says ‘Calculate’. The plot will be drawn with a blackbody curve representing a 300 K body emitting radiation.

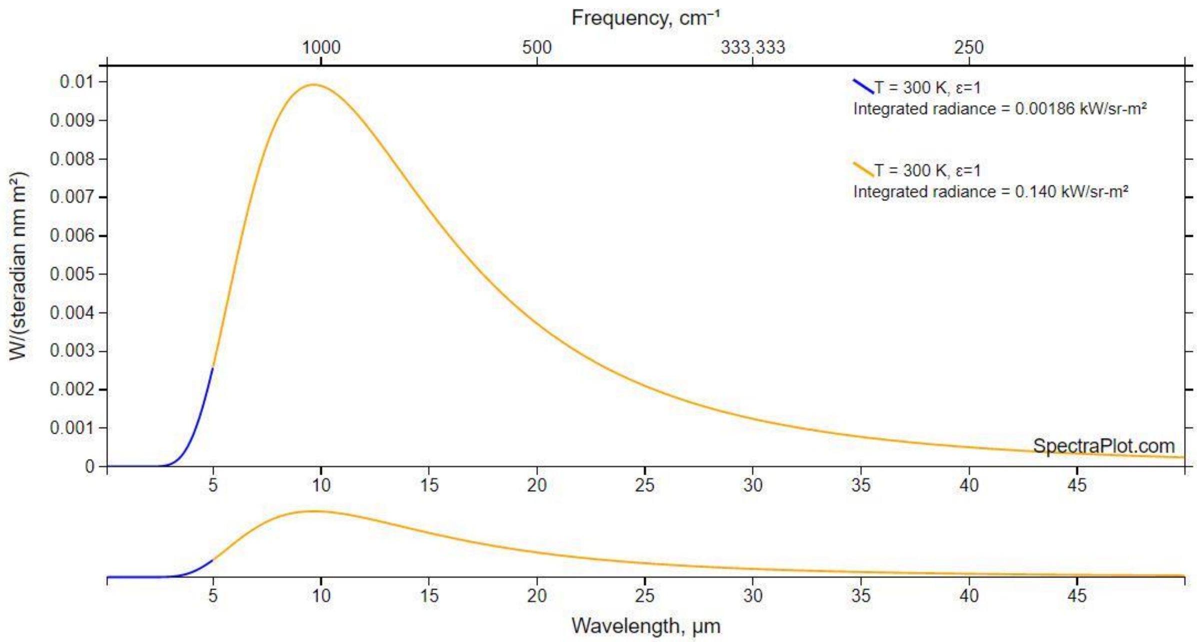


Figure 53 – An example of a mathematically-calculated Planck curve. (Credit *SpectraPlot*)

Question 1: At what wavelength will an object at 300 K emit most of its EM energy? (About 9 microns)

Question 2: What is the temperature of a body that has a peak emission at 28 microns? (About 105 k.)

B-Your smartphone as an astronomical imager

Goal: Students learn how to take astronomical pictures with their smartphone.

Your smartphone is a perfect example of a remote device (spacecraft) that takes images with a camera tuned to specific wavelengths (Red, Green, Blue 'RGB' filters) and then sends the pictures to your laptop via a radio signal (email: Wi-Fi). Then on your laptop you use *Photoshop* (image processing software) to enhance the picture and remove flaws. Almost everything that the JWST does has an analog in some part of this common-day picture taking process.

In this experiment, you will use your smartphone to take pictures of the night sky and see stars invisible to the naked eye! You will follow many of the same steps JWST will, in taking the picture, and transmitting it to a ground station (home!) to process and analyze it.

Required Material:

- ✓ Laptop and access to internet.

Procedure:

Step 1) You will need a recent phone model such as a Galaxy S8 or iPhone 8Plus or others introduced since about 2017 which have faint light capabilities. To see if you phone has this capability, go out some evening just after sunset and use your camera to photograph people, trees or other objects. If your camera has faint-light capability, the photos should look bright as though they were taken in the daytime. If they look black or very grainy, your phone may not work for this experiment. But you can still follow the discussion of what we will do with the picture after it has been taken by your camera. Any subject will do to work with, but a constellation full of stars would be very cool!

Step 2) Check that your camera app has the ability to let you manually change the exposure speed and ISO value. If it does not, download from your app store a camera app that allows tis manual mode.

Step 3) You will need a camera tripod (Cost about $25) and an adaptor for your smartphone (Cost about $15). Attach the adaptor to the camera head, and then clamp your smartphone into the adaptor as shown in the image above.

Step 4) On a dark night where you can see stars in the sky, set up your tripod and smartphone with the exposure set at about 10 seconds and the ISO at 800. Set the delay timer at 5 seconds.

Step 5) Select a portion of the sky that is framed by interesting foreground objects, tree shadows, buildings etc. such as the example shown here.

Step 6) Take an exposure. The delay timer will count down 5 seconds before actually taking the picture. This will help remove vibrations from the tripod and camera that can degrade the photo.

Step 7) Inspect the photo to check that the sky is not washed out with light pollution (called sky fog) and that you can see a number of stars. You may need to take many pictures before you get an interesting photo. Increasing the ISO will improve the number of stars you capture but will also cause the 'dark' sky to start to look grainy. Choose an exposure and ISO that gives you the most stars for the least sky fog.

Step 8) If you are unable to take a good constellation photo, take a photo of some other daytime object instead.

Step 9) When you get a few good pictures, save the image to your phone's gallery. Exit the camera app and open the gallery. Click on the star field photo/s and email them to yourself.

Step 10) On your laptop, open your email server and download the photo/s.

Step 11) In a program like *Photoshop*, load the photo and use the various image enhancement tools to darken the sky and sharpen the star images until you get the 'perfect' photo.

Figure 54 - The constellation Orion taken with an iPhone 6s, with the camera on a tripod and the exposure set at 10 seconds with ISO 400. (Credit: Sten Odenwald).

C- Exploring telemetry math with your smartphone

Goal: Students work with bytes, bites, bandwidth in transmitting images from their camera to their laptop via a Wi-Fi connection.

In this experiment, you will use your smartphone (spacecraft) to download an image to your laptop (Data Analysis Center) and use a program like *Photoshop* (data analysis software) to process the raw image into a nicer one. You will also do the calculations to keep track of the data.

Required Material:

- ✓ Laptop and access to internet.
- ✓ Smartphone

Procedure:

Step 1) Taking the image – A Galaxy S9 phone uses a Sony Exmor IMX333 main camera sensor with a format of 4032x3024 pixels. The sensor takes three images R, G, B simultaneously and saves them as a single RAW file. In this uncompressed format, each R, G, B pixel is recorded in a 14-bit intensity format with 16,384 levels of brightness per color. JPEG files are saved as compressed 8-bit files one per color for 256-levels per color. If you own an android phone, download the *AIDA64* devices app. Open the app and select 'Devices'. The screen will refresh with information about your rear-facing (back) camera. For iOS phones, these is no such convenient app but you can use the above data for the IMX333 sensor. Calculate the size of one uncompressed RAW file and one compressed JPEG file for your camera in both bits and bytes.

Example: RAW=4032x3024 pixels x 14 pits/pixel = 170 megabits or 21.3 megabytes. JPEG=4032x3024 pixels x 8 bits/pixel = 97.5 megabits or 12.2 megabytes.

Step 2) Storing data in the buffer – A Galaxy S9 phone has 4 gigabytes of RAM storage. From Step 1 calculate how many images your phone can store in Raw and JPEG formats.

Example RAW=4000 megabytes x 8 bits/megabyte = 32000 megabits. Then 32000 megabits/170 megabits = 188 RAW images. For JPEGS: 4000 megabytes/12.2 megabytes = 327 JPEG images.

Step 3) Transmission via email – To get images from your phone to your laptop you will probably use a local Wi-Fi network to transfer the images. You will need to know the bandwidth of your Wi-Fi network to calculate the transmission time. On your laptop, first right-click on the **Wi-Fi** icon at the lower right corner area of the screen, then click on *Open Network and Sharing Center*. Next, click on *Wi-Fi connection* or *Hardware Properties*, which will open up the status window that shows you the current *Link Speed (receive/transmit)* in Mbps (megabits per second). From your answers in Step 1, calculate how long it will take to download your RAW and JPES images from the full memory of your smartphone.

Example: The Galaxy S9 has 4 gigabytes of RAM or 32 billion bits. It will take 32 billion / 72 million = 444 seconds or 7.4 minutes to transmit the images.

Step 4) Image processing – Start your laptop image editing app such as *Photoshop, GIMP* or *Pixlr* for example, and open the image you took in Step 1. Use the editor to crop the image, adjust the colors and adjust the image intensity/brightness. Astronomers use a variety of image processing programs such as IDL or programs developed by each mission to handle their own image data.

Step 5) Image analysis – Create a list of all the different things that you can see in your image. This is what astronomers do when they create a statement about what kind of object or objects are appearing in their image: Example, stars, infrared sources, dark clouds, emission nebulae, spiral galaxies.

Step 6) Image interpretation – What do you think is going on in the image? Can you tell a story about what the different image elements are doing? Is there anything unusual about your story? Astronomers create stories such as 'The two galaxies are gravitationally interacting with each other', 'The young stars are ionizing their environments and producing an emission nebula', or 'The dust cloud is being locally heated by embedded proto-stars'.

Step 7) Publication of results - Once you have analyzed and interpreted your image, you will want to share it with your friends and family. Astronomers often will publish the results of their analysis if it supports or seems to refute a prevailing hypothesis or theory about what is being observed.

D-Smartphone thermal imaging in far-red band

Goal: Smartphones can be used as infrared thermal imagers for warm objects that appear to not be emitting light to the human eye.

Smartphone cameras have a red 'R' filter that has unavoidable long-wavelength leakage. This means that although a visible light image may mostly be sensitive to light at wavelengths from 500-650 nanometers, additional light at wavelengths up to 1-micron can also get through and be recorded. Objects with temperatures cooler than 1000 K do not produce much light at wavelengths shorter than 800 nanometers but do produce light at longer wavelengths. An R filter may not be expected to normally see such a cool object but the infrared leak of the filter still makes the camera sensitive to this far-red light. This means that if you took a photo of a hotplate on your stove, it may appear invisible to the eye but its infrared heat can still be detected in the R filter of the camera.

Required Material:

- ✓ Laptop and access to internet.
- ✓ Stove hotplate

Procedure:

Step 1) At night with stray lights shut off, turn on your electric stove or hotplate so that the heating elements are glowing red near maximum heat.

Step 2) With the kitchen pitch-black, start your smartphone camera and select the manual mode so that you can adjust the exposure speed and ISO value.

Step 3) Turn off the hotplate and when you can no longer see the dull red glow of the heating element, take several photos of the spot where the heating element was located. You may have to cycle the stove top on and off briefly so that you can see the area on the camera screen.

Step 4) Repeat steps 1-3 several times with different settings for the ISO and exposure speed. You should probably leave the exposure speed near 1 second and adjust the ISO from 100 to 1000.

Step 5) Email the photo/s to yourself and on your laptop open your mail server and download the pictures to your laptop

Step 6) Open an editing program such as *Photoshop* and adjust each of these photos by using the various tools. A black image can be 'contrast stretched' to reveal faint details. You should see the circular shape of the heating element that was invisible to your eyes when you took the photo.

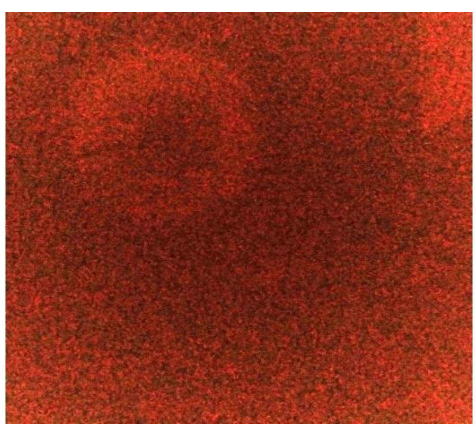


Figure 55 - A *Photoshop*-stretched image of a typical hot plate with a temperature of 375° F (190° C or 460 k). The hotplate is the circular object in the upper left corner. The image was taken with a Galaxy S8 with an exposure of 1/30-sec at ISO 3,200. The graininess is caused by the high ISO number, revealing deflects in the camera imager. (Credit: Sten Odenwald)

E- Infrared radiation from your remote channel selector

Goal: Students explore a common source of infrared light.

The main technology used in home remote controls is infrared (IR) light. The signal between a remote-control handset and the device it controls consists of pulses of infrared light, which is invisible to the human eye but can be seen through a digital camera, video camera or a phone camera. Light from an IR remote control is typically modulated to a frequency not present in sunlight, and the receiver only responds to 0.98-micron light. Smartphone camera designers add an IR-blocking filter into the main 'back camera', which would render the IR emission invisible, however, the front camera 'selfie' does not usually have this blocking filter. The IR emission from the LED will show up as a rather bright spot through the camera display as you press the buttons on the remote. Your camera can photograph this using its normal automatic settings. It is, however, tricky to time the photo so that it catches the LED at its maximum light, which lasts less than 1 second.

<u>Required Material</u>:

- ✓ Laptop and access to internet.
- ✓ TV remote controller

Figure 56 – A common TV remote unit emits an infrared beam. You can't see it with your eye but a photo taken with the selfie camera has enough infrared sensitivity to discern its pulse of light. (Credit: Sten Odenwald)

Procedure:

Step 1) Obtain a working TV remote control.

Step 2) Start your smartphone camera app

Step 3) Point your selfi camera at the front end of the remote.

Step 4) Press the remote a few times while looking at the smartphone screen to see the selfi field of view. You should see a bright spot of light appear for a fraction of a second as you press the remote buttons.

Step 5) Repeat Step 4 but this time try to take a simultaneous selfi picture. This may take a number of trials as you juggle the phone, the control and the camera button. You may need a friend to hold the remote.

F – Exploring non-contact infrared thermometers

Goal: Students explore measuring temperature by detecting the infrared light it emits rather than by direct contact with the surface.

<u>**Required Material**</u>:

- ✓ A non-contact IR thermometer (cost $30.00)

Procedure:

Step 1) Infrared thermometers are available for about $30. Purchase one.

Step 2) Start it up and read its directions for use.

Step 3) Make some practice measurements by measuring some common objects such as an adult's forehead, a child's forehead, an ice cube, a kitchen hot plate on its low setting, and check that the units make sense.

Step 4) Outdoors, place a white sheet of paper on the ground and after a few minutes make a measurement.

Step 5) Replace the white sheet of paper with a black garbage bag or other black surface. After a few minutes make a temperature measurement.

Figure 57 – Measuring the temperature of a surface with a non-contact IR thermometer. This healthy cat has a temperature of 96.3^{o} F and was not at all bothered by the measurement! (Credit: Sten Odenwald).

Question) How do the measurements compare? Which surface is hotter?

Question) What is your explanation for the temperature difference?

G- A simple IR transmitter and receiver

Goals: Students create an inexpensive transmitter and receiver of infrared light analogous to the radio-wave experiment by Heinrich Hertz during the 1890's.

This simple transmitter-receiver demonstrates how electromagnetic radiation at infrared wavelengths can be transmitted and received under controlled conditions. Heinrich Hertz performed a similar experiment by generating radio waves with a spark gap and receiving the radiation with a separate spark gasp system several meters away. The IR transmitter consists of a light-emitting diode (LED) that generates light in the infrared wavelengths near 0.98-microns as a current flows through it. The amount of current is controlled by the potentiometer and can be varied to increase or decrease the infrared light produced by the LED. The receiver consists of a device called a photodiode, which varies its current depending on how much infrared light falls on it. The LED in the receiver varies its brightness depending on how much IR light falls on the photodiode.

Materials:

- 6 - AA batteries
- 2-AA battery holders for 3 batteries (4.5V total) each
- A 100-ohm resistor
- A 1,000-ohm variable potentiometer
- An IR light-emitting diode (LED)
- A photodiode
- An ordinary LED

These components can be purchased from Adafruit.com or Amazon.com

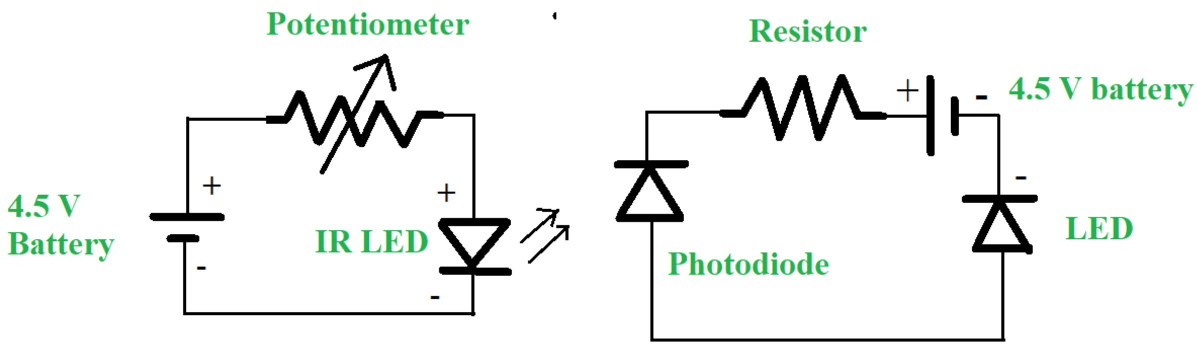


Figure 56 – Circuit diagram for an infrared transmitter and receiver. (Credit: Sten Odenwald).

Construction:

Step 1) Assemble the components in series according to Figure 56. Make sure the polarity of the batteries and LEDs are correct. The battery symbol shows its negative 'flat' end (short segment in symbol) is connected to the negative end (bar) of the diode.

Step 2) With the transmitter circuit powered by connecting the battery, move the receiver circuit so that the photodiode faces the IR LED. The LED pilot lamp in the receiver should light up.

Step 3) Experiment with varying the potentiometer in the transmitter to change the brightness of the receiver LED.

Step 4) Place different materials in the space between the IR Led and the photodiode to see what kinds of materials transmit or block the IR energy.

Step 5) When finished, disconnect the batteries to avoid draining them while storing the system.

H- Exploring Wien's Displacement Law

Goal: Students learn about the mathematical relationship between peak wavelength and temperature for black bodies

<u>Required Material</u>:

- ✓ Laptop and access to internet.

Procedure:

Step 1) Visit one of the black body curve calculators available online

- ✓ https://www.spectraplot.com/blackbody
- ✓ https://demonstrations.wolfram.com/BlackbodySpectrum/

Step 2) On the top of the page, enter 300 in the box for T(K) to select a temperature of 300 K

Step 3) In the windows on the second column, in the top box enter 5 and the bottom enter 50 to select a plot range from 5 to 50-microns.

Step 4) Click the bar to the right that says 'Calculate'. The plot will be drawn with a blackbody curve representing a 300 K body emitting radiation.

Step 5) Record the temperature and the peak wavelength in a table.

Step 6) Create a plot of temperature (horizontal) and peak wavelength (vertical) and

Step 7) Compare your data points against the formula for the Wien's Displacement Law given by $\lambda = 2898/T$.

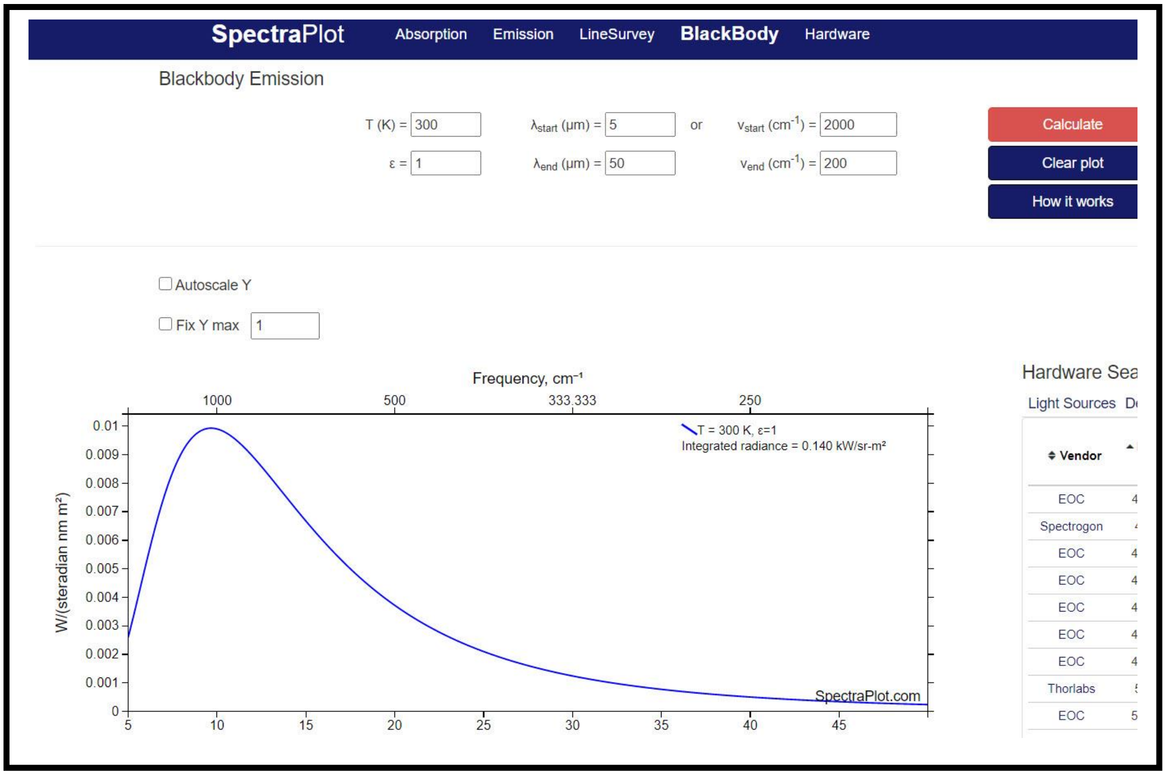


Figure 57 – Screen from *SpectraPlot.com* showing Steps 1-3. (Credit: *SpectraPlot.com*)

I-Exploring Thermal imaging

Goal: Students learn about thermal imaging and pattern recognition

Infrared energy is invisible but the advance of technology has provided a number of means for rendering heat as a visible phenomenon. When one body comes into contact with another, heat energy is exchanged that allows the cooler body to heat up. Heat-sensitive, or thermochromic, paper is coated with a solution of micro-capsules containing a color-changing substance called a leuco dye.

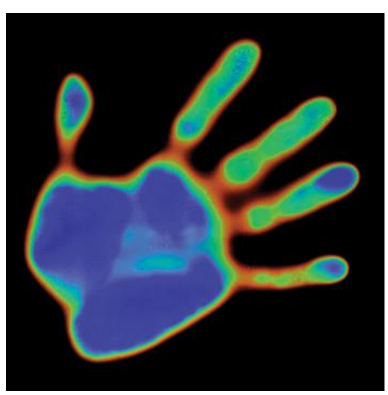


Figure 58 - Color changing liquid crystal with change of temperature. (Credit: Educational Innovations)

Leuco dyes also contain acids which, when exposed to heat, react and become temporarily colorless, reverting to color when cooled. It is often used in fax machines or in cash registers for printing receipts. Liquid crystals also change color with applied heat and are popular as 'mood rings', thermometers and other common applications.

Required Material:

- ✓ Heat-sensitive paper. For example, on Amazon.com you can get *Steve Spangler's Heat Sensitive Paper*, which comes in a package of 100 sheets for $26.00.

Procedure:

Step 1) Obtain a sheet of thermochromic paper (8.5x11) and cut it into four equal rectangles.

Step 2) On each rectangle (5.5 x 4.25 inches) draw a square that is 10cm x 10cm in size. Mark out a set of equal-sized squares that are 2-cm on a side, making a square that has 5x5 cells. This rectangle will serve as a 25-pixel infrared camera.

Step 3) Select a simple geometric object that is at your body temperature and that fits comfortably inside the 5x5 pixel grid.

Step 4) Place the object at the center of the grid and leave it there for a few seconds until it leaves a color-changed impression of its shape.

Figure 59 – Example of an item imaged by its heat. Can you guess what it is? (Credit: Sten Odenwald).

Step 6) Can your partner identify what the object was from its infrared 'heat' signature?

Step 7) Place the paper at different distances from a baseboard or steam-heating element and see how far it can be detected before there is no change in the paper.

J-Infrared mapping

Goal: Students can measure infrared heat energy from invisible or hidden objects.

Infrared cameras are arrays of infrared sensors that measure the intensity of the infrared light in the direction they are pointing. The data from each cell of the array, or pixel, consists of numbers that indicate the intensity of the infrared light. Astronomers use these numbers to create images of an object the same way that your smartphone camera does by assigning colors to the numbers. When these 'colorized' infrared images are overlain on optical photographs, different and sometimes hidden features are revealed that could not be detected with visible light. This experiment demonstrates this principle of hidden infrared images in visual photographs.

Required Material:

- ✓ Thermometer
- ✓ Four ice cube trays.
- ✓ Three paper cups.
- ✓ Food coloring.
- ✓ Coffee stirrers, or spoons.
- ✓ Paper: graph paper, or printed grid to match ice cube tray array
- ✓ Pencil or pen.
- ✓ Colored markers, pencils, or crayons with three colors blue, red and yellow.

Procedure:

Step 1) The four ice cube trays side-by-side give a grid of cells 8 wide by 2 tall. Reproduce this grid on a piece of paper. Create a recognizable, simple pattern by coloring the cells blue, red or yellow.

Step 2) Fill three cups with clear water, with three different temperatures of water: Hot, ice cold, and room temperature.

Step 3) With the food coloring, add a drop of food coloring to each cell to form a second visible pattern that is different from the one selected in Step 1.

Step 4) Using the technique of pinching the lip of the paper cup to make a small pitcher, students carefully fill selected cells of an ice cube tray, some cells with hot, some with cold, and some with lukewarm water following the pattern you chose in Step 1.

Step 3) Measure and record the temperature in each cell, letting the thermometer reading settle for 30 seconds in each cell.

Step 4) Create a false color thermal image, by assigning colors to temperature ranges. For example, a temperature less than 10^{o}C = 50^{o}F could be considered cold, and get a blue color, while 10^o C – 35^o C = 50^o – 90^o F could be considered warm, and get a yellow color, and above 35^o C = 90^o F could be considered hot, and get a red color.

This experiment demonstrates that a visible light image contains one kind of information about an object or scene (Step 3) shown in Figure 60, but an infrared image can often reveal very different information that cannot be seen with visible light (Step 1, Step 4) shown in Figure 61.

Figure 60 - Visible light design using dyes and ice cube trays. (Credit: Paul Mirel/NASA)

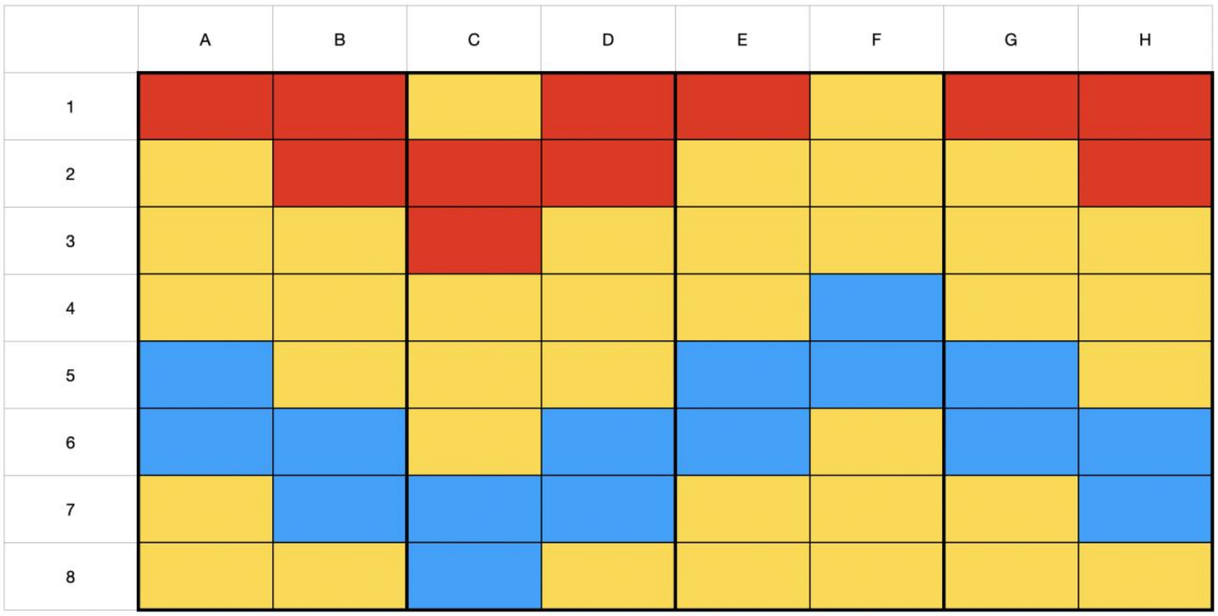


Figure 61 – The hidden thermal pattern created by measuring temperatures in each cell and assigning a color. (Credit: Paul Mirel/NASA)

K - Exploring electromagnetic radiation

Goal: Students explore the transmission of electromagnetic waves in a variety of common situations involving electrical sparks.

Although spark-gap radio transmitters were abandoned in the 1920s, millions of spark gap generators can be found around you inside automobiles. Figure 62 shows the diagram for the ignition circuit in a typical automobile. The sparks in a ‘spark plug’ are nothing more than the sparks generated by Heinrich Hertz in his 1895 experiments. The battery is the standard 12-volt battery in your car. The ignition coil is the large cylindrical device attached to your engine block. The ‘Points’ switch is a rotary device called the distributor. At high speeds, the shaft inside the rotary turns fast and sends out rapid on-off signals to each one of the spark plugs. The timing has to be correct when multiple spark plugs are involved so that they sequentially spark and keep the pistons moving in a smooth motion. The gaps in the spark plugs also have to be clean and narrow enough that a spark can jump between them. The ignition coil operates at 20,000 to 30,000 volts to produce the strong spark discharges needed to ignite the gasoline vapor.

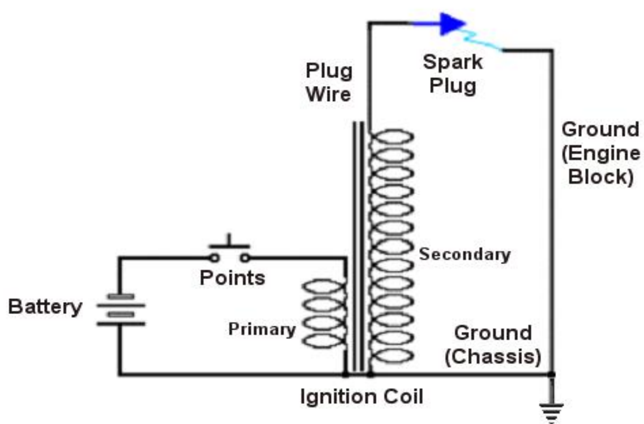


Figure 62 – The electrical circuit for a common automobile spark plug. (Credit: Wikipedia)

The sparks cause broad-band radio noise from 30,000 to 30 million Hz, which can in principle be heard on your car’s radio if you put the car in idle so that the pulses are slow enough to hear individually.

If you hear a clicking noise that increases in tempo as you increase the engine RPMs, it is either alternator or ignition noise caused by the changing frequency of the sparks. A well-tuned car does not have this problem so not all cars you try will have this sparking noise, but you might try the experiment with your lawn mower or motor cycle!

Required Material:

- AM radio
- Access to a car
- CFL lamp

Procedure:

Step 1) Obtain a portable AM radio and tune it to the low end of the band near 530 kHz. With your volume control set to half-scale, you will usually hear faint crackling noises and perhaps the sound from a distant radio station.

Step 2) Inside a car with the engine running on idle (800-1200 RPM) listen for fast, rhythmic pulsations that appear when the car is turned on and stop when the car is off. Also listen for the 'pops' that occur when the ignition switch is turned on producing a spark.

Step 3) Find a compact fluorescent lamp (CFL) in your house and put the radio close to the bulb. You should hear the loud static noise from the sparks used to excite the mercury vapor in the tube.

Step 4) The electrical wiring in your house carries 110 V alternating current (AC). This current flows in one direction during the first half of the cycle then reverses and flows the opposite way in the second half. This movement of electrical current generates an extremely low frequency electromagnetic wave that can be detected using radios that can pick up extremely-low-frequency (ELF) radiation but generally your AM radio will remain silent. Electrical utility companies move electricity with the least possible loss of power through their cables and transformers. To do this, the entire network is tuned to that the vast majority of the energy is at a frequency of 50-60 Hz with very little energy at other frequencies called harmonics. This means that your typical home wiring will not produce electromagnetic radiation at frequencies much above 300 Hz (the 5^{th} harmonic of 60 Hz).

Step 5) High-voltage electrical power lines can be found in many communities. These are suspended on tall towers. Find a location where these wires pass across a roadway or another accessible area. Stand underneath one of these cables. In some situations, you will be able to hear the crackling of the electricity and the hum in the overhead cables.

Figure 63 – Electrical distribution ‘power’ lines near the C&O Canal in Maryland. (Credit: Sten Odenwald).

This hum is produced by the electrical discharges into the air surrounding the cable called corona discharge. The audible noise emitted from high-voltage lines like those in Figure 63 is caused by the discharge of energy that occurs when the electrical field strength on the conductor surface is greater than the 'breakdown strength' (the field intensity necessary to start a flow of electric current) of the air surrounding the conductor. This discharge is also responsible for radio noise, a visible glow of light near the conductor, an energy loss known as corona loss and other phenomena associated with high-voltage lines. These discharges are more frequent when the air is humid because water droplets serve to build up charges on their surfaces that aid in the corona discharge process. To check that such power lines are producing EMR, turn on your AM radio and tune it to 530 kHz. On the ground, as you get within 50-feet of the overhead cable, you should hear a dramatic increase in the noise volume. If you are near large electrical transformers, you will also hear this noise.

L-Spectroscopy with your car radio!

Goal: Students use an ordinary radio to explore basic ideas in spectroscopy.

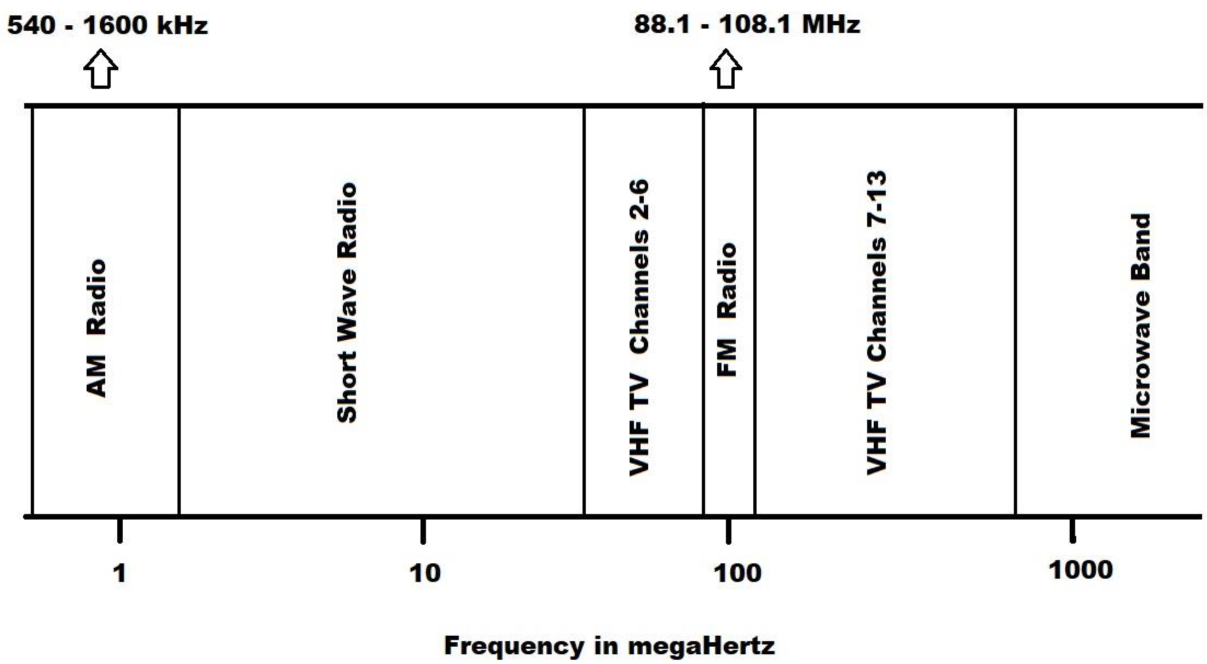


Figure 64 - The location of the AM and FM bands in the radio portion of the EM spectrum. (Credit: Sten Odenwald)

Astronomers use instruments called spectroscopes to 'tune-in' on the light produced by specific elements. These elements produce light at only a small number of specific wavelengths but these are unique to each element and so astronomers look for the fingerprint spectral lines of an element to identify it. JWST will have a spectrometer sensitive to the infrared light that specific kinds of molecules produce such as water, carbon dioxide and ammonia among others.

Your radio can show you how this works by letting you scan across the radio band and note what kinds of radio stations you find from 'molecules' such as country/western; rock-and-roll, news, Spanish language, religion and sports. In between these molecules you will hear radio static and perhaps even some weaker stations too. You can also download a radio app to your smartphone such as *Simple Radio -Live AM FM Radio App*, *Radio App* or *Scanner App*(iOS)/

Figure 65 – Example of a typical AM/FM tuner. (Credit: Sten Odenwald)

Required Material:

- ✓ Car radio or other AM or FM radio with a tuning dial as in Figure 65.

Procedure:

Step 1) Turn on the radio (open the app)

Step 2) Select United States, your state, and a nearby city.

Step 3) Move the AM-band tuner to the start of the radio band

Step 4) Scan the tuner until you arrive at the first station

Step 5) Listen to the station and decide what kind of molecule is this station

Step 6) Note the frequency and enter it in the table on the appropriate row and under Line #1.

Step 7) Scan the tuner to the next station in the band and repeat Steps 5 and 6.

Step 8) Continue this search and table-entry process until you have explored the entire radio band.

Table 7 - Data entry table

Molecule	Line #1	Line #2	Line #3	Line #4	Line #5
Rock n Roll					
Country					
News					
Religion					
Sports					

Question 1) What is the wavelength range of the AM band in meters if the frequency range is 560 to 1,600 kilohertz? w = c/f so w = $3x10^8$ / 560000 = 535meters w = $3x10^8$/1600000=187 meters. The range is from 187 to 535 meters.

Question 2) Which 'molecule' has the most emission lines? Why do you think that is so? What can you tell about the interests of the listeners to the AM band? If there are more 'sports' molecules than others, the listeners prefer to hear more stories about sports because this interests them more than other subjects, and so more stations include sports in their programming.

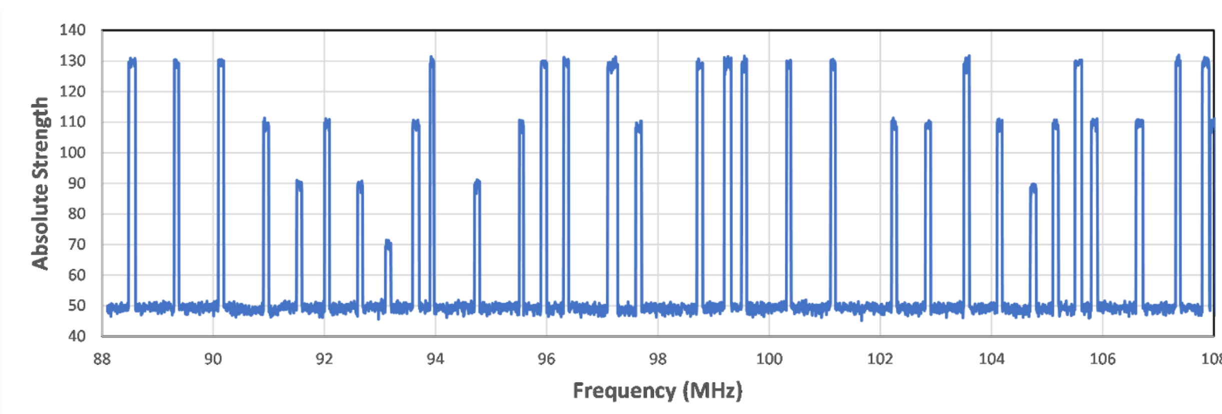


Figure 66 – A simulation of stations detected along the FM band in the Washington DC area. (Credit: Sten Odenwald)

VII – Mathematical Explorations in Infrared Science

Goal: Students explore infrared science through a variety of mathematics problems

Problem 1 – Wavelength and frequency

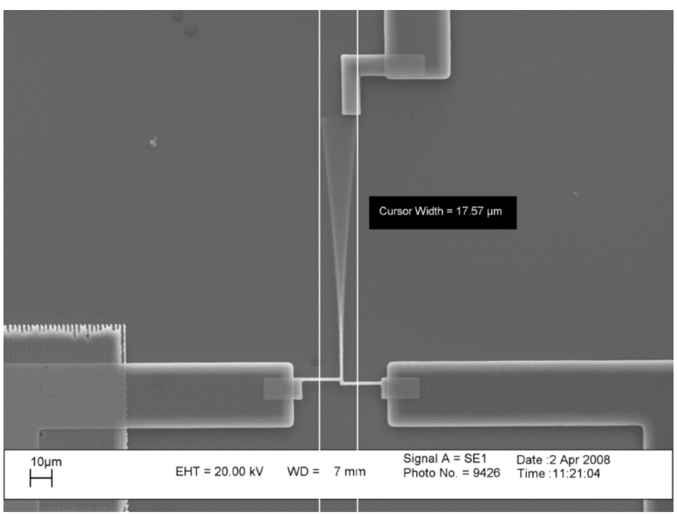
Electromagnetic waves travel at the speed of light so that the wavelength and frequency of these waves are related by the formula $c = \lambda \nu$, where $c = 3 \times 10^8$ m/s, λ is the wavelength in meters and ν is the frequency in Hertz (cycle-per-second). An astronomer is investigating the emissions from a distant nebula at a wavelength of 28-microns in the infrared, and at a frequency of 2.6 gigahertz in the radio spectrum.

A) What is the wavelength of the radio waves in meters?
B) What is the wavelength of the infrared emission in meters?
C) How many times longer are the radio waves than the infrared waves?

Problem 2 – Working with scale models to explore micron units

A-On a scaled drawing, draw a circle to show the sizes of common very small things.

B-Use ruler and scale calculations to work with images in micron units.



Question: What is the length of the microcantilever?

Question: If the longest IR wavelength detected by Webb is 28 microns, how wide is this image in wavelengths?

Problem 3 - Kelvin temperatures and very cold things

The two formulas below show how to switch from degrees-C to degrees-F. Because the Kelvin scale is related to the Celsius scale, we can also convert from Celsius to Kelvin (K) using the equation: **K = 273 + C.** The Celsius and Fahrenheit scales are related according to:

$$\mathbf{F = \frac{9}{5}C + 32}$$

Use these equations to convert between the three temperature scales:

A - 212 F converted to K.

B - 0 K converted to F.

C - +100 C converted to K.

D - -150 F converted to K.

E - -150 C converted to K.

F - Two scientists measure the daytime temperature of the moon using two different instruments. The first instrument gives a reading of +107 C while the second instrument gives + 221 F. A) What are the equivalent temperatures on the Kelvin scale; B) What is the average daytime temperature on the Kelvin scale?

Temp (K)	Event / Object
183	Vostok, Antarctica
160	Phobos
134	Warm Superconductors
95	Titan
90	Liquid oxygen
77	Liquid nitrogen
63	Solid nitrogen
55	Pluto - summer
54	Solid oxygen
50	Quaoar
45	Shadowed crater on moon
40	Star-forming nebula
33	Pluto - winter
20	Liquid nitrogen
19	Bose-Einstein condensate
4	Liquid helium
3	Cosmic background light
2	Liquid helium
1	Boomerang nebula
0	Absolute Zero

Problem 4 – Understanding the Wein Displacement Law

A- At what temperature will an object emit most of its energy at the Webb wavelength of 28 microns?

B- At what wavelength does a human emit most of its heat energy if T= 310 K?

C- A star-forming cloud called CB244 has an embedded young star with a measured temperature of -255 C. At what wavelength will it appear the brightest?

Problem 5 - Why are hot things red?

$$B_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}.$$

The black body formula is given by the formula where h is Planck's Constant (6.6×10^{-34} Joules sec), c is the speed of light (3×10^8 meters/s), k_B is Boltzmann's constant (1.38×10^{-23} Joules/K). Also, λ is the wavelength in meters and T is the temperature in kelvins.

Using differential calculus, take the derivative of B with respect to wavelength λ and find the formula for the maximum emission by setting $dB/d\lambda = 0$ to derive the Wein Displacement Law.

Problem 6 - Stellar Temperature, Size and Power

The amount of power that a star produces in light is related to the temperature of its surface and the area of the star. The hotter a surface is, the more light it produces. The bigger a star is, the more surface it has. When these relationships are combined, two stars at the same temperature can be vastly different in brightness because of their sizes.

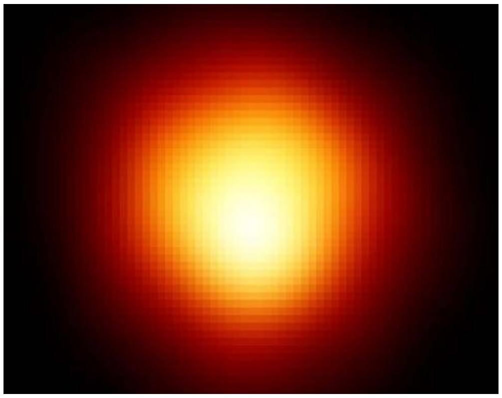


Image: Betelgeuse (Hubble Space Telescope.) It is 950 times bigger than the sun! (Credit A. Dupree (CfA and R. Gilliland (STScI/NASA).

The basic formula that relates stellar light output (called luminosity) with the surface area of a star, and the temperature of the star, is L = A x F where the star is assumed to be spherical with a surface area of $A = 4 \pi R^2$, and the radiation emitted by a unit area of its surface (called the flux) is given by $F = \sigma T^4$. The constant, σ, is the Stefan-Boltzmann radiation constant and it has a value of $\sigma = 5.67 \times 10^{-5}$ ergs/ (cm^2 sec deg^4). The luminosity, L, will be expressed in power units of ergs/sec if the radius, R, is expressed in centimeters, and the temperature, T, is expressed in degrees Kelvin. The formula then becomes,

$$L = 4 \pi R^2 \sigma T^4$$

A - The Sun has a temperature of 5700 Kelvins and a radius of 6.96×10^5 kilometers, what is its luminosity in A) ergs/sec? B) Watts? (Note: 1 watt = 10^7 ergs/sec).

B - The red supergiant Antares in the constellation Scorpius, has a temperature of 3,500 K and a radius of 700 times the radius of the sun. What is its luminosity in A) ergs/sec? B) multiples of the solar luminosity?

C - The nearby star Sirius has a temperature of 9,200 K and a radius of 1.76 times our Sun, while its white dwarf companion has a temperature of 27,400 K and a radius of 4,900 kilometers. What are the luminosities of Sirius-A and Sirius-B compared to our Sun?

<u>**Calculus*:***</u>

D - Compute the total derivative of L(R, T). If a star's radius increases by 10% and its temperature increases by 5%, by how much will the luminosity of the star change if its original state is similar to that of the star Antares? From your answer, can you explain how a star's temperature could change without altering the luminosity of the star. Give an example of this relationship using the star Antares!

Problem 7 - Exploring a Dusty Young Star

This is an image taken by the Spitzer Space telescope in the infrared part of the electromagnetic spectrum. Instead of seeing the light from stars, it sees mainly the light from heated dust grains in space, glowing with temperatures between 100 to 200 K degrees. These bright young stars are found in a rosebud-shaped (and rose-colored) nebulosity known as NGC 7129. The star cluster and its associated nebula are located at a distance of 3300 light-years in the constellation Cepheus. A recent census of the cluster reveals the presence of 130 young stars. The stars formed from a massive cloud of gas and dust. Most stars in our Milky Way galaxy, including our own sun, are thought to form in such clusters. Most of the infrared light comes from a deeply embedded proto-star called FIR-2, which produces 430 times the total power of the sun, with a temperature of about 35 K. We can use this information to estimate the mass of this nebula!

A - A single dust grain is about 0.2 microns in diameter, and has a density of about 2 grams/cm^3. What is the total mass, in grams, of this dust grain if it has a spherical shape?

B - What is the total power produced by this infrared source if the power from the sun is 3.8 x 10^{33} ergs/sec?

C - A single dust grain, 0.2 microns in diameter, at a temperature of 35 K, emits about 7.0 x 10^{-13} ergs/sec of power. How many dust grains are needed to produce the infrared power observed from FIR-2?

D - From your answer to Problem 1 and 3, what is the total mass of dust grains involved in producing the infrared light from FIR-2; A) in grams? B) In units of the sun's mass which is 1.9 x 10^{33} grams?

E - By mass, the interstellar medium consists of 99% gas and 1% dust grains. If the gas within FIR-2 has the same composition, what is the total mass of the interstellar medium within FIR-2 in Solar Masses?

Problem 8 - Deriving the Stefan-Boltzmann Constant

Use integral calculus to derive the Stefan-Boltzmann Law and determine the value of the constant. The intensity of black body radiation from an object at a temperature of T is given by the Planck formula

$$I(\lambda, T) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$

for which the units are watts/m^2/micron. This is a measure of how much power is being emitted by a surface (m^2) over a narrow wavelength range (microns). To determine the total power it is emitting per unit area of surface (called the flux) over its entire electromagnetic spectrum, we have to use integral calculus to sum the radiant flux over all wavelengths, essentially finding the area under the Planck function. Perform this integration.

Problem 9 – A home energy audit via infrared emission

Where is the most heat loss occurring in this house shown in Figure 25? Where would you place more insulation to reduce the loss of heat?

Problem 10 – Understanding filters

Over what wavelength range (bandpass) is the green 'G' filter able to pass more than half of the light in the visible spectrum from 400 to 700 nanometers?

Problem 11 - Working with Filters

A particular filter only passes 43% of the light at a wavelength of 0.625 microns. If the incident radiation has an intensity of I(0.625)= 9.3×10^{-5} watts/m^2/Hz, what is the final intensity of the light seen through the filter?

Problem 12 - The Launch of the Mars Science Laboratory (MSL) in 2011

This sequence of stills was obtained from a YouTube.com video of the launch of MSL by United Space Alliance available at **http://www.youtube.com/watch?v=0cxsvVBemHY**

Students use a sequence of launch images to determine the Atlas V's launch speed and acceleration. By determining the scale of each image, they estimate average speeds during the first 4 seconds after lift-off. This sequence shows the launch of the MSL mission from the Kennedy Space Center Launch Complex 49 on November 27, 2011 at 10:02 EST. The four images were taken, from bottom to top, at times 10:02:48 EST, 10:02:50 EST, 10:02:51 EST and 10:02:52 EST. At the distance of the launch pad, the width of each image is 400 meters.

A - With the help of a millimeter ruler, what is the scale of each image in meters/mm?

B - For each image, what is the distance between the bottom of the image and the base of the rocket nozzle for the Atlas V rocket in each scene?

C – What is the estimated distance from the base of the launch pad to the rocket nozzle in each image?

D – From the time information, what is the average speed of the rocket between A) Image 1 and 2? B) Image 2 and 3? C) Image 3 and 4?

E – From the speed information in Problem 4, what is the average acceleration between: A) Image 1 and Image 3? B) Image 2 and Image 4?

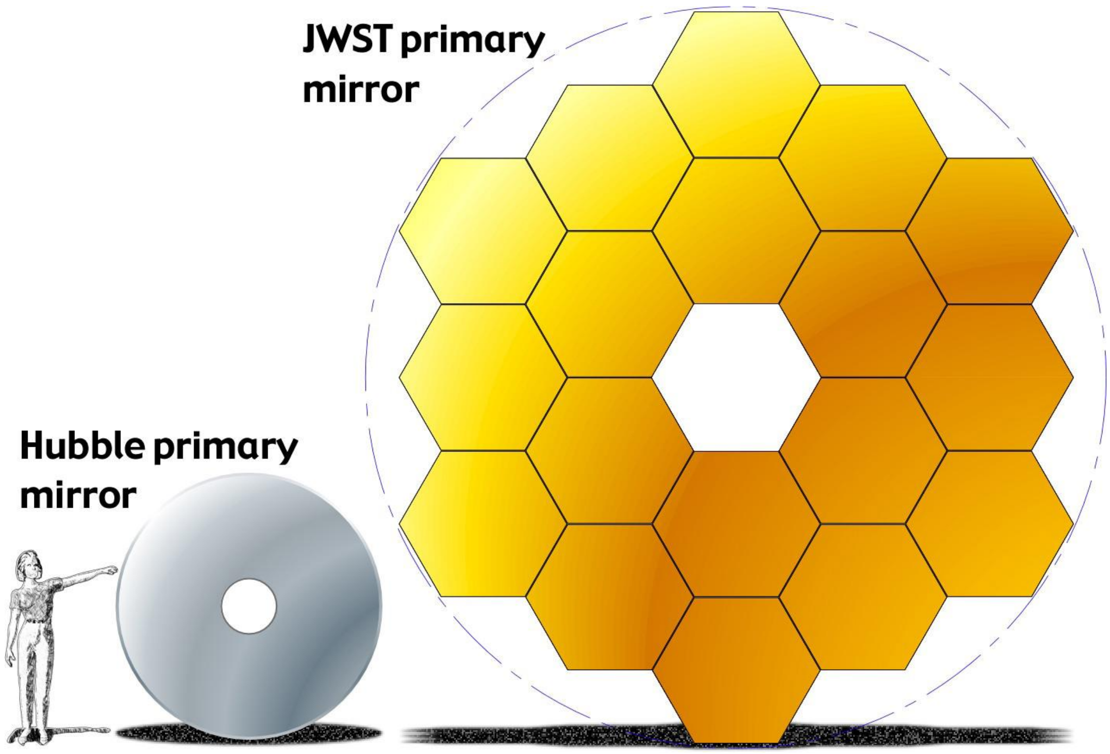
F – Graph the height of the rocket versus the time in seconds since launch.

G – Graph the speed of the rocket versus time in seconds after launch. For the time, use the midpoint time for each speed interval.

H – Graph the acceleration of the rocket versus time in seconds after launch. For the time, use the midpoint time for each acceleration interval.

Problem 13 - Scaling Up the Webb Space Telescope Mirror

Students learn about the Webb Space Telescopes segmented mirror and determine the area of the mirror along with scaled up versions of this mirror using the formula for the area of a hexagon, and the properties of tiling a surface with hexagons.



https://www.jwst.nasa.gov/content/observatory/ote/mirrors/index.html

The James Webb Space Telescope, to be launched by NASA in 2016, is a telescope designed to explore galaxies and stars that formed soon after the Big Bang. Its unique design features a large mirror that consists of 18 hexagonal tiles; each tile is its own mirror. Many of the largest telescope mirrors now being built for ground-based observatories use the hexagonal 'segmented' design. A single 1-meter-wide hexagon, replicated dozens of times, is a lot easier to make than a single large mirror! Suppose that in the problems below, the length of a side of the hexagon is L = 0.76 meters. New mirror designs are created from the Webb Space Telescope design by adding enough mirror tiles to complete a new outer ring. For example, the Webb Space Telescope mirror consists of two complete rings of hexagonal tiles.

A- Using the sketch above as a guide, how many tiles will be in the assembled mirror if 1, 2 or 3 additional rings of hexagonal tiles are added?

B - What is the total area of each mirror design for 1,2 or 3 added rings if the area of a single hexagon is

$$A = \frac{3}{2}\sqrt{3}\, L^3$$

Compared to the Webb Space Telescope design of 18 tiles, by what factor do the three new mirror designs exceed the Webb Space Telescope collecting area?

Problem 14 - The Hexagonal Tiles in the Webb Space Telescope Mirror

The James Webb Space Telescope, to be launched by NASA in 2021, is a telescope designed to explore galaxies and stars that formed soon after the Big Bang. Its unique design features a large mirror that consists of 18 hexagonal tiles; each tile is its own mirror. The 18 mirror tiles work together to form a single large mirror to collect faint star light. An important feature of a telescope mirror is its surface area. The more surface area a mirror has, the more light it can collect. To make faint stars and galaxies appear bright enough to study in detail, mirrors with large collecting areas are needed.

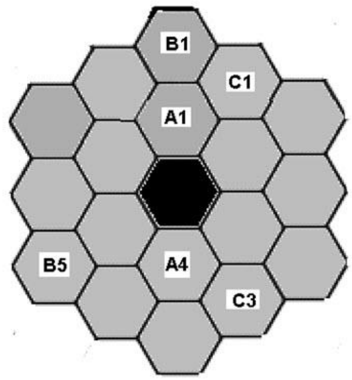
A – For a regular hexagon with a side length of L, how many equilateral triangles with a side length of L can you fit into one hexagon?

B – From the drawing of the Webb Space Telescope Mirror, how many equilateral triangles are formed by the 18 hexagonal tiles?

Problem 15 - Six-fold Symmetry and the Webb Space Telescope Mirror

Students learn about the Webb Space Telescopes segmented mirror and its rotational 6-fold symmetry due to tiling with hexagons. They identify groups of tiles that have identical optical properties

The Webb Space Telescope segmented mirror consists of 18 hexagonal mirror tiles assembled to make a larger mirror just over 6 meters in diameter. The placement of these tiles is not random, however. Tiles located at a specific distance from the center of the mirror are manufactured with exactly the same optical properties. For example, in the drawing, each of the inner ring of 6 tiles has an identical 'twin' to each of the other tiles in this ring.



Another property of regular hexagons is that, when you rotate them by 60 degrees, the pattern looks identical to the one you started with. Suppose you labeled one of the six sides, Side A, and placed it at the top of the pattern. If you rotate the hexagon in steps of 60 degrees, it will take exactly 6 shifts to bring Side A back to the top of the pattern. The assembled Webb Space telescope mirror shows this same pattern among the tiles.

A - Using the tile labeling shown above, find all other tiles that follow 6-fold symmetry and label them using the same scheme. How many classes can you identify, and how many tiles per class?

B - If you were to add one more ring of hexagonal tiles to the outer edge of the Webb Space Telescope mirror, how many different kinds of mirror tiles would there be, and in each class, how many identical mirror tiles would be present with the same optical properties?

Problem 16 – Gold film on the JWST primary mirror.

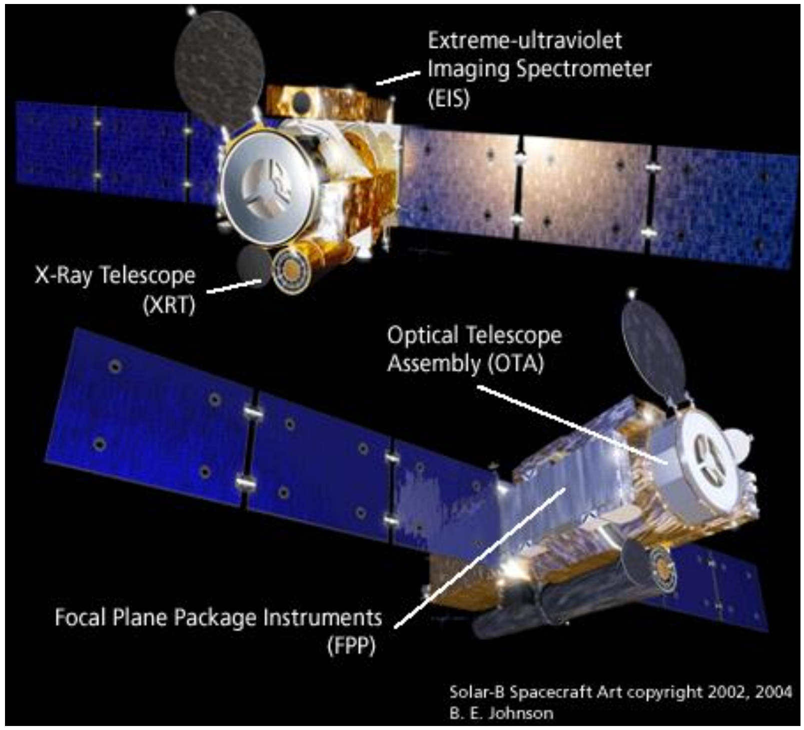
The thickness of the gold coating on the primary mirror is 0.1-microns. The area covered is 25 square meters. If the density of gold is 19.3 gm/cm^3

A) How many grams of gold were used to coat the mirror?

B) If the commercial cost of gold is about $59,400 per kilogram, how much did the coating cost?
C) The mass of a golf ball is about 46 grams. How many golf balls-worth of gold is in the mass of the mirror coating?

Problem 17 - Hinode Satellite Power

The Hinode satellite, launched to study the sun in 2005, weighs approximately 700 kg (dry) and carries 170 kg of gas for its steering thrusters, which help to maintain the satellite in a polar, sun-synchronous orbit for up to two years. The satellite has two solar panels (blue) that produce all of the spacecraft's power. The panels are about 4 meters long and 1 meter wide, and are covered on both sided by solar cells.



A - What is the total area of the solar panels covered by solar cells in square centimeters?

B - If a solar cell produces 0.03 watts of power for each square centimeter of area, what is the total power produced by the solar panels when facing the sun? Can the satellite supply enough power to operate the experiments which require 1,150 watts?

C - Suppose engineers decided to cover the surface of the cylindrical satellite with solar cells instead. If the satellite is 4 meters long and a diameter of 1 meter, how much power could it produce? Can the satellite supply enough power to keep the experiments running, which require 1,150 watts?

Problem 18 - Digital Camera Math

Digital cameras are everywhere! They are in your cell phones, computers, iPads and countless other applications that you may not even be aware of. In astronomy, digital cameras were first developed in the 1970s to replace and extend photographic film techniques for detecting faint objects. Digital cameras are not only easy to operate and require no chemicals to make the images, but the data is already in digital form so that computers can quickly process the images.

Commercially, digital cameras are referred to by the total number of pixels they contain. A '1 megapixel camera' can have a square-shaped sensor with 1024x1024 pixels. This says nothing about the sensitivity of the camera, only how big an image it can create from the camera lenses. Although the largest commercial digital camera has 80 megapixels in a 10328x7760 format, the largest astronomical camera developed for the Large Synoptic Survey Telescope (LSST) uses 3200 megapixels (3.2 gigapixels)!

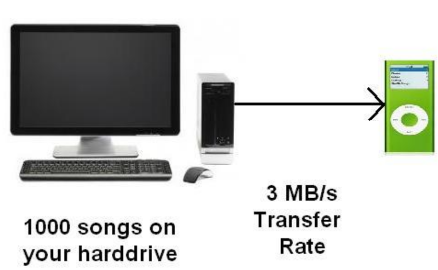
A – An amateur astronomer purchases a 6.1-megapixel digital camera. The sensor measures 20 mm x 20 mm. What is the format of the CCD sensor, and about how wide are each of the pixels in microns?

B – Suppose that with the telescope optical system, the entire full moon will fit inside the square CCD sensor. If the angular diameter of the moon is 1800 arcseconds, about what is the resolution of each pixel in the camera?

C – The LSST digital camera is 3.2 gigapixels in a 10328x7760 format. If the long side of the field covers an angular range of 3.5 degrees, what is the angular resolution of this CCD camera in arcseconds/pixel?

Problem 19 - Exploring the InSight Lander Telemetry Data Flow

Transferring digital data from place to place takes time. Like water flowing into a lake, the faster it flows the more rapidly the lake fills up and overflows. With computer data, we have a similar problem. You have probably had to do this yourself many times. Each time you copy your playlist from your laptop to your portable music player, you will have to wait a certain length of time. The transfer rate is fixed, so the more songs you want to transfer the longer you have to wait. Here's how this works!



A - Suppose you want to transfer 1000 songs from your laptop collection to your music Player. Each 4-minute song takes up 4 megabytes on the laptop, and the cable link from your computer to your Player can handle a transfer rate of 3 million bytes/second. How many minutes does it take to transfer all your songs to the Player?

B - How long will it take to fill up the buffer with data?

C - How long will be required to transmit the buffer data to Earth during each 2-hour transmission cycle?

D – The receiver on Earth can be scheduled to contact the Lander as often as once every 2 hours. How large a buffer would you need so that you could gather as much data as 4 megabytes/sec over 2 hours? How long does it take the instruments to gather this much data?

Problem 20 - Advanced Unit Conversions

Students work with more unit conversions and use them to solve a series of practical problems in science and solar energy.

1 Astronomical Unit = 1.0 AU = 1.49x10^8 kilometers

1 parsec = 3.26 lightyears = 3x10^{18} centimeters = 206,265 AU

1 watt = 10^7 ergs/sec

1 Star = 2x10^{33} grams

1 year = 365.25 days

1 day = 24.0 hours

A – Convert 11.3 square feet into square centimeters.

B – Convert 250 cubic inches into cubic meters.

C – Convert 1000 watts/meter^2 into watts/foot^2

D – Convert 5 miles into kilometers.

E – Convert 1 year into seconds.

F – Convert 1 km/sec into parsecs per million years.

G - A house is being fitted for solar panels. The roof measures 50 feet x 28 feet. The solar panels cost \$1.00/cm^2 and generate 0.03 watts/cm^2. A) What is the maximum electricity generation for the roof in kilowatts? B) How much would the solar panels cost to install? C) What would be the owners cost for the electricity in dollars per watt?

H – A box of cereal measures 5 cm x 20 cm x 40 cm and contains 10,000 Froot Loops. What is the volume of a single Froot Loop in cubic millimeters?

I – In city driving, a British 2002 Jaguar is advertised as having a gas mileage of 13.7 liters per 100 km, and a 2002 American Mustang has a mileage of 17 mpg. Which car gets the best gas mileage?

J – The Space Shuttle used 800,000 gallons of rocket fuel to travel 400 km into space. If one gallon of rocket fuel has the same energy as 5 gallons of gasoline, what is the equivalent gas mileage of the Space Shuttle in gallons of gasoline per mile?

K – The length of an Earth-day increases by 0.0015 seconds every century. How long will a day be in 3 billion years from now?

L – The density of matter in the Milky Way galaxy is 7.0 x 10^{-24} grams/cm^3. How many stars are in a cube that is 10 light years on a side?

M – At a speed of 300,000 km/sec, how far does light travel in miles in 1 year?

Problem 21 - The Most Important Equation in Astronomy

There are many equations that astronomers use to describe the physical world, but none is more important and fundamental to the research that we conduct than the one to the left! You cannot design a telescope, or a satellite sensor, without paying attention to the relationship that it describes.

In optics, the best-focused spot of light that a perfect lens with a circular aperture can make, is limited by the diffraction of light. The diffraction pattern has a bright region in the center called the Airy Disk. The diameter of the Airy Disk is related to the wavelength of the illuminating light, λ, and the size of the circular aperture (mirror, lens), given by D. When λ and D are expressed in the same units (e.g centimeters, meters), R will be in units of angular measure called radians (1 radian = 57.3 degrees). $R = 1.22\ \lambda/D$

You cannot see details with your eye, with a camera, or with a telescope, that are smaller than the Airy Disk size for your particular optical system. The formula also says that larger telescopes (making D bigger) allow you to see much finer details. For example, compare the top image of the Apollo-15 landing area taken by the Japanese Kaguya Satellite (10 meters/pixel at 100 km orbit elevation: aperture = about 15cm) with the lower image taken by the LRO satellite (1.0 meters/pixel at a 50km orbit elevation: aperture = 0.8 meter). The Apollo-15 Lunar Module (LM) can be seen by its 'horizontal shadow' near the center of the image.

A - The Senator Byrd Radio Telescope in Green Bank West Virginia with a dish diameter of D=100 meters is designed to detect radio waves with a wavelength of λ= 21-centimeters. What is the angular resolution, R, for this telescope in A) degrees? B) Arc minutes?

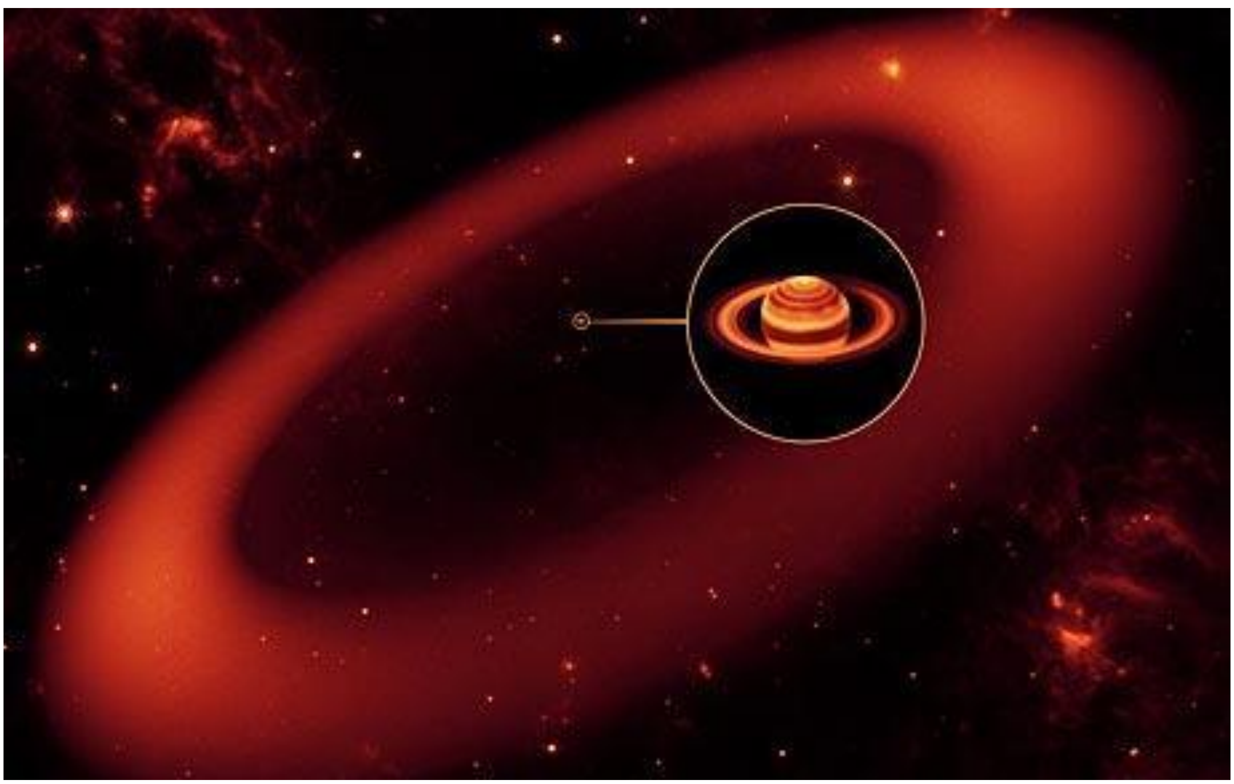
B - The largest, ground-based optical telescope is the D = 10.4-meter Gran Telescopio Canaris. If this telescope operates at optical wavelengths (λ = 0.00006 centimeters wavelength), what is the maximum resolution of this telescope in A) microradians? B) milliarcseconds?

C- An astronomer wants to design an infrared telescope with a resolution of 1-arcsecond at a wavelength of λ=20 micrometers. What would be the diameter of the mirror?

Problem 22 - Spitzer Telescope Discovers New Ring around Saturn!

Students calculate the volume of the ring and compare it to the volume of Earth to check a news release figure that claims 1 billion Earths could fit inside the new ring.

The thin array of ice and dust particles lies at the far reaches of the Saturnian system. The ring was very diffuse and did not reflect much visible light but the infrared Spitzer telescope was



able to detect it. Although the ring dust is very cold -316 F it shines with thermal 'heat' radiation. No one had looked at its location with an infrared instrument until now. "The bulk of the ring material starts about 6.0 million km from the planet, extends outward about another 12 million km, and is 2.6 million km thick. The newly found ring is so huge it would take 1 billion Earths to fill it." (CNN News, October 7, 2009) Artist rendering of the new ice ring around Saturn detected by the Spitzer Space Telescope. "This is one supersized ring," said one of the authors, Professor Anne Verbiscer, an astronomer at the University of Virginia in Charlottesville. Saturn's moon Phoebe orbits within the ring and is believed to be the source of the material.

A - What is the formula for the area of a circle with a radius R from which another concentric circle with a radius r has been subtracted?

B - What is the volume of the region defined by the area calculated in Problem 1 if the height of the volume is h?

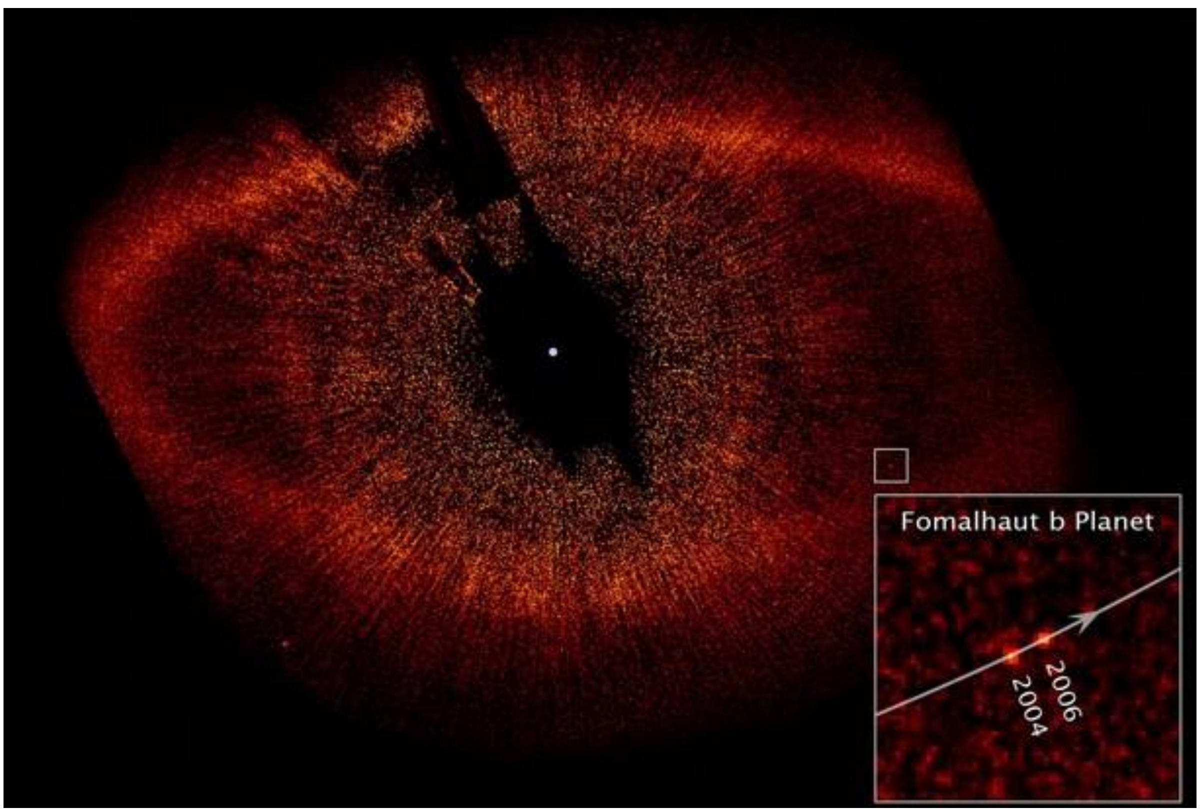
C - If r = 6×10^6 kilometers, R = 1.2×10^7 kilometers and h = 2.4×10^6 kilometers, what is the volume of the new ring in cubic kilometers?

D - The Earth is a sphere with a radius of 6,378 kilometers. What is the volume of Earth in cubic kilometers?

E - About how many Earths can be fit within the volume of Saturn's new ice ring?

F - How does your answer compare to the Press Release information? Why are they different?

Problem 23 - Hubble Sees a Distant Planet



The bright star Fomalhaut, located in the constellation Piscis Austrinus (The Southern Fish) is only 25 light years away. It is 2000° K hotter than the Sun, and nearly 17 times as luminous, but it is also much younger: Only about 200 million years old. Astronomers have known for several decades that it has a ring of dust (asteroidal material) in orbit 133 AU from the star and about 25 AU wide. Because it is so close, it has been a favorite hunting ground in the search for planets beyond our solar system. In 2008 such a planet was at last discovered using the Hubble Space Telescope. It was the first direct photograph of a planet beyond our own solar system.

In the photo, the dusty ring can be clearly seen, but photographs taken in 2004 and 2006 revealed the movement of one special 'dot' that is now known to be the star's first detected planet. The small square on the image is magnified in the larger inset square in the lower right to show the location of the planet in more detail.

A – The scale of the image is 2.7 AU/millimeter. If 1.0 AU = 150 million kilometers, how far was the planet from the star in 2006?

B – How many kilometers had the planet moved between 2004 and 2006?

C – What was the average speed of the planet between 2004 and 2006 if 1 year = 8760 hours?

D – Assuming the orbit is circular, with the radius found from Problem 1, about how many years would it take the planet to make a full orbit around its star?

Problem 24 – Working with expanding space

The fact that our universe is expanding was predicted by Einstein's theory of general relativity by the mathematical development of 'Big Bang' cosmology, but was also detected for the first time by astronomer Edwin Hubble. The expansion of the universe means that, although the locations of distant galaxies remain at fixed coordinate positions, the scale of the coordinate grid is increasing. Distances between galaxies increase, not because the galaxies are moving, but because the space between them is stretching or dilating. From any vantage point, this appears as though galaxies are rushing away at speeds the increase as they become more distant. This is called Hubble's Law and represented by the formula V = H d where V is the recession speed in km/s and d is the distance to the galaxy in megaparsecs, where 1 megaparsec = 3,260,000 light years. The current observed value for H is 69 km /s/mpc.

A) An astronomer uses a spectrograph to measure the spectral lines of hydrogen in the galaxy NGC 2342, and finds that they have been shifted by an amount corresponding to a doppler speed of 5,690 km/s. What is the distance to that galaxy in light years?
B) Astronomers use the redshift parameter, z, to indicate the wavelength change caused by the expansion of space according to $z = (\lambda - \lambda r)/\lambda r$ where λ is the observed wavelength in Angstroms and λr is the wavelength of the same spectral line at rest in the laboratory. The recession speed of the object is then V = cz where c = the speed of light 300,000 km/s.

An absorption feature of calcium usually has a wavelength of 3934 Å, but it is observed in a galaxy to have a wavelength of 4002 Å. What is the recession speed?

C) The formula in Part B is only accurate if the recession speeds are less than 10% the speed of light. For redshifts of z=1 or greater the relativistic formula below is used. In 1999, astronomers have identified nearly 1000 galaxies with z > 2 and more than 50 galaxies with z > 5. The most remote galaxy at that time was HDF 4-473.0 with a redshift of z=5.6. What is the recession speed for this infant galaxy?

$$V = \left[\frac{(z+1)^2-1}{(z+1)^2+1}\right] c$$

D) The redshift factor z is also a measure of the amount by which the universe's scale has expanded according to $1+z = R_0/R$ where R is the scale of the universe at the time the light was emitted, and R_0 is the scale of the universe today, represented by R_0=1.0. In 2021 astronomers discovered the farthest known galaxy GN-z11 with a redshift of z=11.4. What was the scale of the universe when its light was first emitted 13.4 billion years ago?

Problem 25 – How big is the universe?

Our universe has been expanding for 13.4 billion years, which means that objects that were once close together can now be billions of light years apart. Because the expansion of space is not limited by the speed of light, the distances between objects can change faster than the speed of light as space dilates. The objects are not actually traversing the space between them to reach their large separations. This means that our universe contains regions close enough to us that light could have traversed the distance in the current age of the universe of 13.4 billion years. This region of the universe surrounding us is called the Visible Universe and contains all objects that we now observe. There is a far-vaster universe of galaxies and stars beyond this limit for which we have not as yet received their light since they were formed in the Big Bang. General relativity lets us determine the distances to these objects today, called the comoving distance, from the approximate formula below:

$$d(gLY) = 8.5\ln(z) + 11.7$$

A) The galaxy GN-z11 was recently discovered at a redshift of z=11.4. What is the actual distance to this object today in billions of light years (gLY)?

B) What is the light travel distance to this galaxy if its light was emitted 13.3 billion years ago?

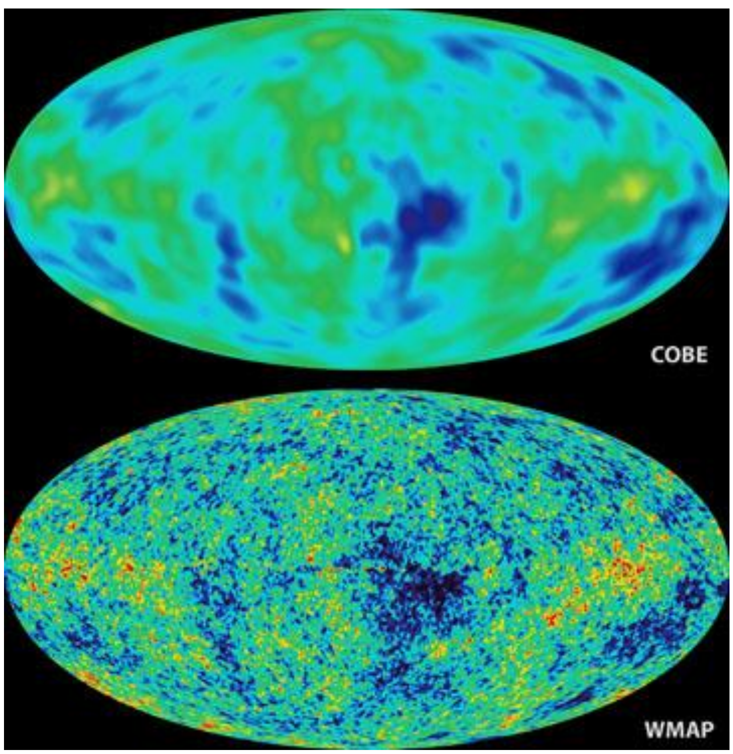
C) At the present time, what fraction of the current volume of space is occupied by our visible universe based on the distance to GN-z11?

Problem 26 – Distance and look-back time.

The images we see of distant objects are as they were long ago because of the finite speed of light. For very distant galaxies, this light-travel time is called the 'look-back' time because you are looking back at what an object looked like when light was first emitted by it to form the image you are seeing. Because of the expansion of space as light travels across it, the amount of travel time is related to the redshift of the object being observed. The lookback time is exactly calculated from a cosmological model, but for the observed properties of our universe, it can be approximated by the equation **T(gyr) = 2.1ln(z)+8.6**. What is the lookback time for the galaxy GN-z11 at a redshift of z=11.4?

Problem 27 - Seeing the Distant Universe

Students calculate the angular sizes and scales of distant objects to study how different sized telescopes see details with varying degrees of clarity.



The JWST uses a very large mirror to enable astronomers to see distant objects more clearly. This is a very important goal of this telescope since astronomers know so little about how objects look far from Earth. In the universe, viewing objects at a great distance also means that you are seeing them as they were long ago.

The scientific mission of the JWST is to study how galaxies like the Milky Way formed when the universe was only a few million years old compared to its current age of over 13 billion years.

The image above shows the sky imaged by the COBE satellite with a 7-degree resolution (top) compared to the WMAP satellite with a 1/2-degree resolution (bottom). Note the increased detail with WMAP.

The sky is measured in angular units (degrees, minutes, seconds) but we would actually like to know how many kilometers a given angular measurements corresponds to. The simple formula below relates size to distance and angular width:

$$L = \frac{\theta}{206265} d$$

where θ is the angular diameter in arcseconds, d is the distance to the object in light years and L is the actual diameter of the object in light years. Note that for closer objects, L and d will both be in units of kilometers.

The smallest feature that the human eye can see on the moon has an angular width of about 2 arcminutes or 120 arcseconds. The smallest feature that the JWST Near Infrared Camera (NIRcam) can see clearly has a width of about 0.032 arcseconds per pixel.

A - What is the width of the smallest feature that the human eye can see on the moon at the distance of Earth, d = 384,000 kilometers?

B - How far, d, would a planet the size of Earth (L = 12,800 km) have to be in order for the Webb Space Telescope to just see it (θ = 0.032 arcseconds)?

C - Suppose that the most distant object that can be detected by the Webb Space Telescope is located 13 billion light years from Earth. What would be the minimum diameter of this object, L, at the maximum resolution of the telescope?

Problem 28 - Webb Space Telescope: Detecting dwarf planets

One of its research goals will be to detect new dwarf planets beyond the orbit of Pluto. In this problem, students use three functions to predict how far from the sun a body such as Pluto could be detected, by calculating its temperature and the amount of infrared light it emits.

In 2021, the new Webb Space Telescope will be launched. This telescope, designed to detect distant sources of infrared 'heat' radiation, will be a powerful new instrument for discovering distant dwarf planets far beyond the orbit of Neptune and Pluto. Scientists are already predicting just how sensitive this new infrared telescope will be, and the kinds of distant bodies it should be able to detect in each of its many infrared channels. This problem shows how this forecasting is done.

A - The angular diameter of an object is given by the formula:

$$\theta(R) = 0.0014\frac{L}{R} \text{ arcseconds}$$

Create a single graph that shows the angular diameter, θ(R), for an object the size of dwarf planet Pluto (L=2,300 km) spanning a distance range, R, from 30 AU to 100 AU, where 1 AU (Astronomical Unit) is the distance from Earth to the sun (150 million km). How big will Pluto appear to the telescope at a distance of 90 AU (about 3 times its distance of Pluto from the sun)?

B - The temperature of a body that absorbs 40% of the solar energy falling on it is given by

$$T(R) = \frac{250}{\sqrt{R}}$$

where R is the distance from the sun in AU. Create a graph that shows θ(R) vs R for objects located in the distance range from 30 to 100 AU. What will be the predicted temperature of a Pluto-like object at 90 AU?

C - A body with an angular size θ(R) given in arcseconds emits 40% of its light energy in the infrared and has a temperature given by T(R) in Kelvins. Its brightness in units of Janskys, F, at a wavelength of 25 microns (2500 nanometers) will be given by:

$$F(T) = \frac{120000}{(e^{x} - 1)} \theta^2 \text{ Janskys} \qquad \text{where} \quad x = \frac{720}{T(R)}$$

From the formula for θ(R) and T(R), create a curve F(R) for a Pluto-like object. If the Webb Space Telescope cannot detect objects fainter than 4 nanoJanskys, what will be the most distant location for a Pluto-like body that this telescope can detect? (Hint: Plot the curve with a linear scale in R and a log10 scale in F.)

Problem 29 - The Cosmological Redshift - Changing the light from a galaxy.

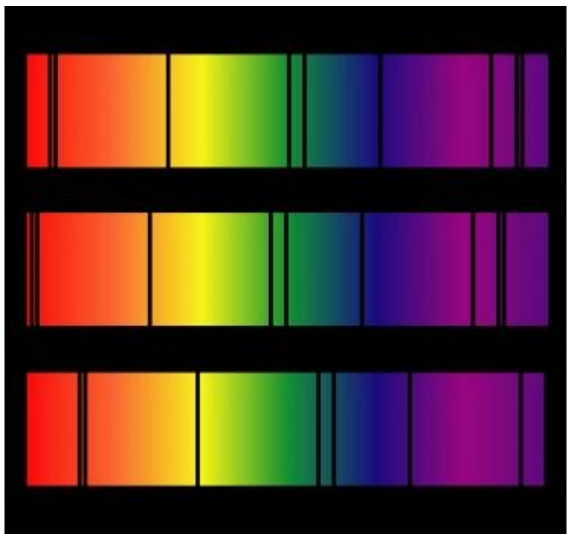


Figure - Top: Normal 'dark' spectral line positions at rest. Middle: Source moving away from observer. Bottom: Source moving towards observer.

We have all heard the sound of an ambulance siren as it passes-by. The high-pitch as it approaches is replaced by a low-pitch as it passes by. This is an example of the Doppler Shift, which is a phenomenon found in many astronomical settings as well, except that instead of the frequency of sound waves, it's the frequency of light waves that is affected.

The figure shows how the wavelengths of various atomic spectral lines normally found in the top locations, are shifted to the red (long wavelength; lower pitch) for a receding source, and to the blue (short wavelength; higher pitch) for an approaching source of light. For very distant galaxies, the effects of curved space cause the wavelengths of light to be increasingly red-shifted as the distance to Earth increases. This is a Doppler-like effect, but it has nothing to do with the speed of the galaxy or star, but on the changing geometry of space over cosmological distances.

Just as in the Doppler Effect, where we measure the size of the Doppler 'red'-shift in terms of the speed of the object emitting the sound waves, for distant galaxies we measure their redshifts in terms of the cosmological factor, z. Close-by galaxies have z-values much less than 1.0, but very distant galaxies can have z=6 or higher. Observing distant galaxies is a challenge because the wavelengths where most of the light from the galaxy are emitted, are shifted from visible wavelengths near 500 nanometers (0.5 microns) to much longer wavelengths. This actually makes distant galaxies very dim in the visible spectrum, but very bright at longer infrared wavelengths. For example, for redshifts of z = 3, the maximum light from a normal galaxy is shifted to a wavelength of λ = 0.5 microns x (1+z) = 2.0 microns!

A - The Webb Space Telescope, Mid-Infrared Instrument (MIR) can detect galaxies between wavelengths of 5.0 and 25.0-microns. Over what redshift interval can it detect normal galaxies like our Milky Way?

B - An astronomer wants to study an event called Reionization, which occurred between 5.0 < z < 7.0. What wavelength range does this correspond to in normal galaxies?

C - The NIRcam is sensitive to radiation between 0.6-5.0 microns, the MIR instrument range is 5.0 to 25.0 microns, and the Fine Guidance Sensor-Tunable Filter Camera detects light between 1 to 5 microns. Which instruments can study the Reionization event in normal galaxies?

VIII – Answer key

Problem 1 – Wavelength and frequency

A) Answer: $\lambda = c/\nu$ so $\lambda = 3\times10^8/2.6\times10^9$ = **1.2-meters**.

B) Answer: 28 microns = 28 micrometers = 28×10^{-6} meters = **2.8×10^{-5} meters.**

C) Answer: Radio/IR = 1.2-meters/2.8×10^{-5} meters = **43000 times**.

Problem 2 – Working with scale models to explore micron units

Question: Answer: With a millimeter ruler to determine the image scaling in microns per millimeter, the microcantilever has a length of **12.3 microns**.

Question: Answer: The width of the image is about 300 microns/28 microns = **11 wavelengths**

Problem 3 - Kelvin Temperatures and Very Cold Things

A - First convert to C: C = 5/9 (212 - 32) = +100 C. Then convert from C to K: K = 273 + 100 = **373 Kelvin**

B - 0 = 273 + C so C = -273 degrees. Then convert from C to F: F = 9/5 (-273) + 32 = **-459 Fahrenheit**.

C - K = 273 + 100 = **373 Kelvin**.

D - C = 5/9 (-150 - 32) = -101 C. Then convert from Centigrade to Kelvin: K = 273 - 101 = **172 Kelvin**.

E - K = 273 + (-150) = **123 Kelvin**

F - A) 107 C becomes K = 273 + 107 = **380 Kelvins.** 221 F becomes C = 5/9 (221 - 32) = 105 C, and so K = 273 + 105 = **378 Kelvins**. B) Answer: (380 + 378)/2 = **379 Kelvins**. C) Answer: **Because the degrees are in the same units in the same measuring scale so that the numbers can be averaged.** Note: Students may recognize that in order to average +107 C and +221 F they could just as easily have converted both temperatures to the Celsius scale or the Fahrenheit scale and then averaged those temperatures. You may challenge them to do this, and then compare the averaged values in the Celsius, Fahrenheit and Kelvin scales. They should note that the final answer will be the same as 379 Kelvins converted to F and C scales using the above formulas.

Problem 4 – Understanding the Wein Displacement Law

A- T=103 K.

B- At what wavelength does a human emit most of its heat energy if T= 310 K? Answer: λ = 9.3 microns

C- T = 18 K.

Problem 5 - Why are hot things red?

Let $U = A\lambda^{-5}$ then $dU/d\lambda = -5A\lambda^{-6}$ where $A = 2hc^2$ and

$$(hc/k) = (6.67\times10^{-34})(3\times10^{8})/(1.38\times10^{-23}) = 14329 \text{ for } \lambda \text{ in microns}$$

Let $V = e^{(14329/(\lambda T))} - 1$ then $dV/d\lambda = -14329/(\lambda^2 T)\ e^{14329/(\lambda t)}$

Then use the quotient rule: $d/d\lambda\,(U/V) = 1/V\ dU/d\lambda - U/V^2\ dV/d\lambda$

to get $dU/d\lambda - U/V\ dV/d\lambda = 0$

Then by substitution

$$-5A\lambda^{-6} - [A\lambda^{-5}]/[e^{(14329/(\lambda T))} - 1] \times (-14329/(\lambda^2 T)\ e^{14329/(\lambda t)}) = 0$$

$- 5 A \lambda^{-6} [e^{(14329/(\lambda T))} – 1] + A \lambda^{-5} (14329 / (\lambda^2 T) e^{14329/(\lambda t)}) = 0$

$5 [e^{(14329/(\lambda T))} – 1] - (T\lambda)^{-1} (14329 e^{14329/(\lambda t)}) = 0$

Let X = 14329/ (λ T) then we get a simpler equation to solve:

$5(e^x - 1) - x e^x = 0$ (Equation 1)

This transcendental equation, when solved, will give the location of the extrema for the Planck Function, however, it cannot be solved exactly. You will need to program a graphing calculator or use an Excel spreadsheet to find the value for X that gives, in this case, the maximum value of the Planck Function.

The table shows some trial-and-error results, and a convergence to approximately X = 4.965. The formula for the peak wavelength is then

4.965 = 14394 / (λ T) or **λ = 2899 / T**

This is similar to the function derived by fitting the tabulated data.

X	Eq. 1
1	5.873127
2	17.16717
3	35.17107
4	49.59815
4.5	40.00857
4.9	8.428978
4.95	2.058748
4.96	0.703752
4.965	**0.015799**

Problem 6 - Stellar Temperature, Size and Power

A - We use L = 4 (3.141) R^2 (5.67 x 10^{-5}) T^4 to get L (ergs/sec) = 0.00071 $R(cm)^2$ $T(K)^4$ then, A) L(ergs/sec) = 0.00071 x (696,000 km x 10^5 cm/km)^2 (5700)^4 = **3.6 x 10^{33} ergs/sec**. B) L(watts) = 3.6 x 10^{33} (ergs/sec)/ 10^7 (ergs/watt) = **3.6 x 10^{25} watts**.

B - A) The radius of Antares is 700 x 696,000 km = **4.9 x 10^8 km**. L(ergs/sec) = 0.00071 x (4.9 x 10^8 km x 10^5 cm/km)^2 (3500)^4 = **2.5 x 10^{38} ergs/sec**. B) L(Antares) = (2.5 x 10^{38} ergs/sec) / (3.6 x 10^{33} ergs/sec) = **69,000 L(sun).**

C - Sirius-A radius = 1.76 x 696,000 km = **1.2 x 10^6 km**

L(Sirius-A) = 0.00071 x (1.2 x 10^6 km x 10^5 cm/km)^2 (9200)^4 =**7.3 x 10^{34} ergs/sec**
L = (7.3 x 10^{34} ergs/sec) / (3.6 x 10^{33} ergs/sec) = **20.3 L(sun).**
L(Sirius-B) = 0.00071 x (4900 km x 10^5 cm/km)^2 (27,400)^4 = **9.6 x 10^{31} ergs/sec**
L(Sirius-B) = 9.6 x 10^{31} ergs/sec / 3.6 x 10^{33} ergs/sec = **0.027 L(sun).**

D - From a basic theorem in differential calculus, for a multi-variable function, the total derivative is related to the 'partial' derivatives in each independent variable so that for L(R,T) we have

$$d\,L(R,T) = \frac{\partial L}{\partial R} dR + \frac{\partial L}{\partial T} dT$$

which gives $dL(R,T) = 8\pi R \sigma T^4 \; dR + 16\pi R^2 \sigma T^3 \; dT$

To get percentage changes, divide both sides by $L = 4\pi R^2 \sigma T^4$

$$\frac{dL}{L} = \frac{8\pi R \sigma T^4}{4\pi R^2 \sigma T^4} dR + \frac{16\pi R^2 \sigma T^3}{4\pi R^2 \sigma T^4} dT$$

Then dL/L = 2 dR/R + 4 dT/T so for the values given, dL/L = 2 (0.10) + 4 (0.05) = **0.40**
The star's luminosity will increase by 40%.

Since dL/L = 2 dR/R + 4 dT/T, we can obtain no change in L if 2 dR/R + 4 dT/T = 0. This means that 2 dR/R = - 4 dT/T and so, - 0.5 dR/R = dT/T. **The luminosity of a star will remain constant if, as the temperature decreases, its radius increases.**

Example. For Antares, its original luminosity is 2.5 x 10^{38} ergs/sec. If you increase its radius by 10% from 4.9 x 10^8 km to 5.4 x 10^8 km, its luminosity will remain the same if its temperature is decreased by dT/T = 0.5 x 0.10 = 0.05 which will be 3500 x 0.95 = 3,325 K so L(ergs/sec) = 0.00071 x (5.4 x 10^8 km x 10^5 cm/km)^2 (3325)^4 = **2.5 x 10^{38} ergs/sec**

Problem 7 - Exploring a Dusty Young Star

A - Diameter = 0.2 microns, radius= 0.1-microns, and Mass = Volume x Density = 4/3 π R^3 x Density = 4/3 x 3.14 x (0.1 microns x 10^{-4} cm/micron)^3 x 2.0 = **8.4 x 10^{-15} grams**

B - Power = 430 x 3.8 x 10^{33} ergs/sec = **1.6 x 10^{36} ergs/sec**

C - Number = (1.6 x 10^{36} ergs/sec) / (7.0 x 10^{-13} ergs/sec/dust grain) = **2.3 x 10^{48} dust grains**

D - Answer: A) 8.4 x 10^{-15} grams / dust grain x 2.3 x 10^{48} dust grains = **1.8 x 10^{34} grams**. B) 1.8 x 10^{34} grams / 1.9 x 10^{33} grams = **9.5 Solar Masses.**

E - 100 x 9.5 = **950 Solar Masses**. The actual dust mass has been estimated as about 6 solar masses by C. Eiroa (AA 1998, v. 335, p. 243. and Muzerolle, et al, 2004, ApJ Suppl. V. 154, p.379.) The largest uncertainty in the problem is the size of the dust grain and the infrared radiation power emitted by the dust grain. This problem assumed that the dust grain size is typical of what is found for interstellar dust grains. But in the environment of the protostar, dust grain sizes are expected to vary due to grain growth. Also, the reflectivity (albedo) of the dust grain depends on its composition and size in a complex way.

Problem 8 - Deriving the Stefan-Boltzmann Constant

$F(T) = \int_0^\infty I(\lambda, T) d\lambda$

$$F(T) = \int_0^\infty \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} d\lambda$$

$$F(T) = 2\pi hc^2 \int_0^\infty \frac{1}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} d\lambda$$

Let $U = \frac{hc}{\lambda kT}$

Then $dU = -\frac{hc}{kT} \lambda^{-2}\, d\lambda$

so $d\lambda = -\frac{kT}{hc} \lambda^2\, dU$

which by substituting $\lambda^2 = \left(\frac{hc}{kT}\right)^2 \frac{1}{U^2}$

gives you $d\lambda = -\frac{kT}{hc} \left(\frac{hc}{kT}\right)^2 \frac{1}{U^2}\, dU$ also $\frac{1}{\lambda^5} = \left(U \frac{kT}{hc}\right)^5$

so, we have after substitution of U for λ

$$F(T) = -2\pi hc^2 \int_0^\infty \left(U \frac{kT}{hc}\right)^5 \frac{1}{e^U - 1} \frac{kT}{hc} \left(\frac{hc}{kT}\right)^2 \frac{1}{U^2}\, dU$$

which simplifies to

$$F(T) = \frac{2\pi k^4 T^4}{h^3 c^2} \int_0^{\infty} U^3 \ \frac{1}{e^U - 1} \ dU$$

What happened to the negative sign? The wavelength and frequency scales run opposite to each other so the (-) sign is a result of this axis directionality, which is immaterial to the area under each curve. We could also have elected to work in frequency units where $\nu = c/\lambda$ and we would have arrived at the same definite integral to solve. In fact, this is a more customary way to set up the problem with $I(\nu,T)$ rather than $I(\lambda,T)$. The calculation would be the same but with no negative sign appearing in the definition for dU in the above substitutions.

The integral can be solved by using a Table of integrals such as https://en.wikipedia.org/wiki/Lists_of_integrals

The integral has a value of $\frac{\pi^4}{15}$

So that the final equation becomes $F(T) = \frac{2\pi k^4}{h^3 c^2}\frac{\pi^4}{15} T^4$

So that for $\sigma = \frac{2\pi^5 k^4}{15 h^3 c^2} T^4$ we have the Stefan-Boltzmann Law $F(T) = \sigma\ T^4$

but where we can now evaluate the Stefan-Boltzmann constant in terms of known constants so for $h = 6.62\times10^{-34}$ Joules Kelvins, $k = 1.38\times10^{-23}$ J K^{-1} c= 3.0×10^8 m/s we have $\sigma = 5.67\times10^{-8}$ W $m^{-2}K^{-4}$.

Problem 9 – A home energy audit via infrared emission

Where is the most heat loss occurring in this house shown in Figure 25? Answer: **Through the windows.**

Where would you place more insulation to reduce the loss of heat? Answer: **Use special, insulated 'thermopane' glass that traps heat inside the house.**

Problem 10 – Understanding filters

Over what wavelength range (bandpass) is the green 'G' filter able to pass more than half of the light in the visible spectrum from 400 to 700 nanometers? Answer: **Between about 475 and 575 nanometers.**

Problem 11 - Working with Filters

I = 0.43 x 9.3×10^{-5} watts/m^2/Hz = **4.0×10^{-5} watts/m^2/Hz**.

Problem 12 - The Launch of the Mars Science Laboratory (MSL) in 2011

A - Width = 69 mm, so scale = 400 m/69 mm = **5.8 meters/mm**

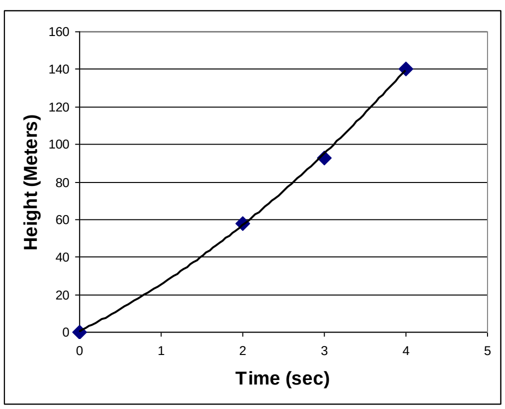
B - 8mm, 18mm, 24mm and 32mm so using the scale of the image, the actual distances are **46m, 104m, 139m and 186 meters**.

C – Take the differences in the measurements relative to the first image at the moment of launch to get: h1 = 46m-46m = **0m**, h2=104m-46m = **58m**, h3=139m-46m = **93 m** and h4 =186m-46m = **140 m**.

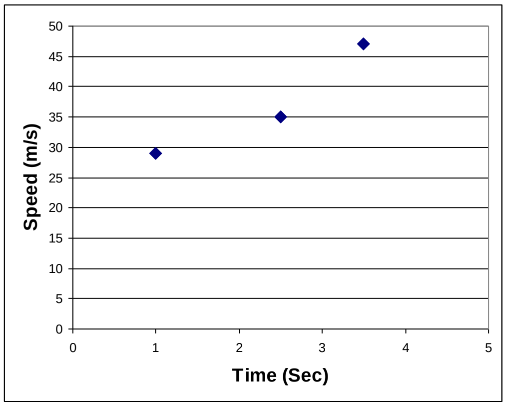
D – A) v= distance/time, v1 = (58m-0m)/2sec = **29m/sec** B) v2 =(93m-58m)/1sec = **35 m/sec**, C) v3=(140m-93m)/1sec = **47 m/sec**.

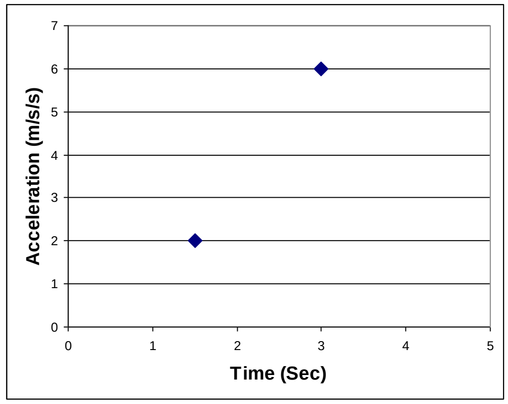
E – A) a1 = (v2-v1)/3sec = (35-29)/3 = 6/3 = **2 m/sec^2**. B) a2 = (V3-v2)/2sec = (47-35)/2sec = **6 m/sec^2**.

F – Graph the height of the rocket versus the time in seconds since launch.



G – For the first speed, the two height measurements are made at T=0 and T=2, so the speed V1 will be plotted at the midpoint time: T=(2-0)/2 = **1 sec**





H – Right Above.

Problem 13 - Scaling Up the Webb Space Telescope Mirror

A- From the shaded rings indicated below: One additional ring (green) = 18 + 18 = **36 tiles**. Two rings = 36 + 24 = **60 tiles**. Three rings = 60 + 30 = **90 tiles.**

B - The hexagon area is A = 1.5 (1.732) $(0.76)^2$ = 1.5 $meters^2$. Webb Space Telescope Mirror: 18 tiles, Area = 18 x 1.5 = 27 $meters^2$. One additional ring: Area = 36 x 1.5 = **54 meters**^{2}. Two additional rings: Area= 60 x 1.5 = **90 meters**^{2}. Three additional rings: Area = 90 x 1.5 = **135 meters**^{2}. The areas increase by factors of **2.0, 3.3 and 5.0 times** the area of the Webb Space Telescope design.

Problem 14 - The Hexagonal Tiles in the Webb Space Telescope Mirror

A –A total of six.

B –Students may draw the triangles inside each of the 18 hexagons and then count them, or can recognize that (6 triangles in each hexagon) x 18 hexagons = **108** equilateral triangles.

Problem 15 - Six-fold Symmetry and the Webb Space Telescope Mirror

A - There are exactly three different tile classes, A, B and C, and 6 tiles per class**.**

B - There are 6 unique mirror classes (A, B, C, D, E, F) and six identical mirrors per class. Each mirror class is at a unique distance from the center of the mirror and has the same optical properties. This means that A1 can be interchanged with A5, but that A1 cannot be swapped for B1, C1 etc.

Problem 16 – Gold film on the JWST primary mirror.

A) Mass = volume x density. The volume is 0.1-micron x 25 m^2. Converting into centimeters we get V= $1.0 \times 10^{-5} \times 25 \times 10^4$ = 2.5 cubic centimeters. Density = 19.3 gm/cm^3, then Mass = 2.5 x 19.3 = **48.3 grams**. B) The cost is 0.0483 kg x $59400/kg = **$2,869**. C) The gold equals about **1 golf ball in mass.**

Problem 17 - Hinode Satellite Power

A - The surface area of a single panel is 4 meters x 1 meter = 4 square meter per side. There are two sides, so the total area of one panel is 8 square meters. There are two solar panels, so the total surface area covered by solar cells is 16 square meters. Converting this to square centimeters: 8 square meters x (10,000 cm^2/m^2) = **80,000 cm^2**

B -Only half of the solar cells can be fully illuminated at a time, so the total exposed area is 40,000 cm^2. The power produced is then: Power = 40,000 cm^2 X 0.03 watts/cm^2 = **1,200 watts. Yes, the satellite solar panels can keep the experiments running, with 50 watts to spare!**

C - Surface area of a cylinder = $2 \pi R^2 h$ so for the satellite S = 2 x (3.14) (0.5 meters) ^{2} (4 meters) = 6.28 square meters. Only half of the solar cells can be illuminated, so the usable area is 3.14 square meters or 31,400 square centimeters. The power produced is 31,400 x 0.03 **= 942 watts. No, the satellite cannot keep the experiments running. They require an extra 1,100-942 = 158 watts.**

Problem 18 - Digital Camera Math

A – This is a square array, so s^2 = 6100000 pixels and so s = 2469 pixels. The format is **2469 x 2469 pixels**. Since the width of a side is 20 mm, each pixel is 0.020 meters/2469 = 8.1×10^{-6} meters or **8.1 microns on a side.**

B – 1800 arcseconds/2469 pixels = **0.7 arcseconds /pixel**.

C – 1 degree = 3600 arcseconds, so 3.5 degrees = 12600 arcseconds. Then 12600 arcseconds/10328 pixels = **1.2 arcseconds/pixel.**

Problem 19 - Exploring the InSight Lander Telemetry Data Flow

A - 4000 megabytes x (1 second/ 3 megabytes) = 1333 seconds or about **22 minutes**.

B - The data enters the buffer at 50 megabytes/hr and the buffer contains 500 megabytes, so it can store data for 500 MBytes/(50Mbytes/hr) = **10 hours**.

C - In 2 hours at a data rate of 50 megabytes/hr you have 100 megabytes stored in the buffer. At a transmission rate of 4 megabytes/sec it takes 100 Mbytes/(4 Mbytes/sec) = **25 seconds** to transmit the100 megabytes from the buffer to Earth.

D – 2 hours equals 2 x 3600 = 7200 seconds. At a transmission rate of 4 Mbytes/sec, this equals 7200 sec x 4 Mbytes/sec = 28,800 Megabytes or **28.8 Gigabytes.** The instruments gather 50 Megabytes/hour, so it would take them 28,800 Megabytes/(50 MB/hr) = 576 hours or 24 days to gather this much data. What this says is that if you had a buffer this large (28.8 gigabytes) it could store 24 days of data from InSight and only take 2 hours to transmit to Earth. The danger of waiting so long to transmit data (every 24 days) is that something could happen to the lander and you would lose all this data! That's why scientists try to download their data as often as possible

Problem 20 - Advanced Unit Conversions

A – 11.3 x (12 inches/foot)x(12 inches/foot) x (2.54 cm/1 inch)x(2.54 cm/1 inch) = **10,500 cm^2**

B – 250 $inch^3$ x $(2.54\ cm/inch)^3$ x $(1\ meter/100\ cm)^3$ = **0.0041 m^3**

C – 1000 $watts/meter^2$ x $(1\ meter/39.37\ inches)^2$ x $(12\ inches/foot)^2$ = **93.0 $watts/ft^2$**

D – 5 miles x (5280 feet/mile) x (12 inches/foot)x(2.54 cm/inch)x(1 meter/100 cm) x(1 km/1000 meters) **= 8.0 km**

E – 1 year x (365.25 days/year)x(24 hours/day)x(60 minutes/hr)x(60 seconds/minute) = **31,558,000 seconds.**

F – 1 km/sec x (100000 cm/km)x(3.2×10^7 seconds/year) x (1 parsec/ 3.1×10^{18} cm) x (1,000,000 years/1 million years) = **1.0 parsec/million years**

G - A) Area = 50 feet x 28 feet = 1400 ft^2. Convert to cm^2: 1400 x $(12\ inch/foot)^2$ x $(2.54\ cm/1\ inch)^2$ = 1,300,642 cm^2. Maximum power = 1,300,000 cm^2 x 0.03 $watts/cm^2$ = **39.0 kilowatts**. B) 1,300,000 cm^2 x \$1.00 /cm^2 = **\$1.3 million** C) \$1,300,000 / 39,000 watts = **\$33 /watts**.

H – Volume of box = 5 x 20 x 40 = 4000 cm^3. This contains 10,000 Froot Loops, so each one has a volume of 4,000 cm^3/10,000 loops = 0.4 cm^3/Loop. Converting this into cubic millimeters: 0.4 cm^3 x $(10\ mm/1\ cm)^3$ = **400 mm^3/Loop**.

I – Convert both to kilometers per liter. Jaguar = 100 km/13.7 liters = 7.3 km/liter. Mustang = 17.0 x (1 km/0.62 miles) x (1 gallon/3.78 liters) = 7.3 km/liter. **They both get similar gas mileage under city conditions**.

J – 400 km x (0.62 miles/km) = 250 miles. Equivalent gallons of gasoline = 800,000 gallons rocket fuel x (5 gallons gasoline/1 gallon rocket fuel) = 4,000,000 gallons gasoline, so the 'mpg' is 250 miles/4000000 = 0.000063 miles/gallon or **16,000 gallons/mile**.

K – 0.00015 sec/century x (1 century/100 years) x 3 billion years = 4,500 seconds or 1.3 hours. The new 'day' would be 24h – 1.3 = **22.7 hours long**.

L –First convert to grams per cubic parsec: 7.0 x 10^{-24} grams/cm^3 x (3.1 x 10^{18} cm/parsec$)^3$ = 2.0 x 10^{32} grams/pc^3. Then convert to Stars/pc^3: 2.0 x 10^{32} grams/pc^3 x (1 Star/2 x 10^{33} grams) = 0.1 Stars/pc^3. Then compute the volume of the cube: V = 10x10x10 = 1000 light years^3 = 1000 light years^3 x (1 parcsec/3.26 light years$)^3$ = 28.9 Parsecs^3. Then multiply the density by the volume: 0.1 Stars/pc^3 x (28.9 Parsecs^3) = **3.0 Stars in a volume that is 10 light years on a side**.

M – 300,000 km/sec x (3.1 x 10^7 sec/year) = 9.3 x 10^{12} km. Then 9.3 x 10^{12} km x (0.62 miles/km) = **5.8 trillion miles**.

Problem 21 - The Most Important Equation in Astronomy

A - First convert all numbers to centimeters, then use the formula to calculate the resolution in radian units: λ = 21 centimeters, D = 100 meters = 10,000 centimeters, then R = 1.22 x 21 cm /10000 cm so R = 0.0026 radians. There are 57.3 degrees to 1 radian, so A) 0.0026 radians x (57.3 degrees/ 1 radian) = **0.14 degrees**. And B) There are 60 arc minutes to 1 degree, so 0.14 degrees x (60 minutes/1 degrees) = **8.4 arcminutes.**

B - R = 1.22 x (0.00006 cm/10400 cm) = 0.000000069 radians. A) Since 1 microradian = 0.000001 radians, the resolution of this telescope is **0.069 microradians**. B) Since 1 radian = 57.3 degrees, and 1 degree = 3600 arcseconds, the resolution is 0.000000069 radians x (57.3 degrees/radian) x (3600 arcseconds/1 degree) = 0.014 arcseconds. One thousand milliarcsecond = 1 arcseconds, so the resolution is 0.014 arcsecond x (1000 milliarcsecond / arcsecond) = **14 milliarcseconds.**

C- From R = 1.22 λ/D we have R = 1 arcsecond and λ = 20 micrometers and need to calculate D, so with algebra we rewrite the equation as D = 1.22 λ/R. Convert R to radians: R = 1 arcsecond x (1 degree/3600 arcsecond) x (1 radian / 57.3 degrees) = 0.0000048 radians. λ = 20 micrometers x (1 meter/1,000,000 micrometers) = 0.00002 meters. Then D = 1.22 (0.00003 meters)/(0.0000048 radians) **= 5.1 meters**.

Problem 22 - Spitzer Telescope Discovers New Ring of Saturn!

A - The area of the large circle is given by πR^2 minus area of small circle πr^2 equals $\mathbf{A = \pi (R^2 - r^2)}$

B - Volume = Area x height so $\mathbf{V = \pi (R^2 - r^2) h}$

C - $V = \pi (R^2 - r^2) h$ = (3.141) [(1.2 x $10^7)^2$ - (6.0 x $10^6)^2$] 2.4 x 10^6 = **8.1 x 10^{20} km^3**

Note that the smallest number of significant figures in the numbers involved is 2, so the answer will be reported to two significant figures.

D - Volume of a sphere $V = 4/3 \pi R^3 = 1.33 \times (3.14) \times (6.378 \times 10^3)^3 =$ **1.06×10^{12} km^3**

Note that the smallest number of significant figures in the numbers involved is 3, so the answer will be reported to three significant figures.

E - Divide the answer in Problem 3 by Problem 4: 8.1×10^{20} km^3 / (1.06×10^{12} km^3) = **7.6×10^8 times**

F - The Press Releases say 'about 1 billion times' because it is easier for a non-scientist to appreciate this approximate number. If we rounded up 7.6×10^8 times to one significant figure accuracy, we would also get an answer of '1 billion times'.

Problem 23 - Hubble Sees a Distant Planet

A – The distance from the center of the ring (location of star in picture) to the center of the box containing the planet is 42 millimeters, then 42 x 2.7 AU/mm = 113 AU. Since 1 AU = 150 million km, the distance is 113 x 150 million **= 17 billion kilometers.**

B– On the main image, the box has a width of 4 millimeters which equals 4 x 2.7 = 11 AU. The inset box showing the planet has a width of 36 mm which equals 11 AU so the scale of the small box is 11AU/36 mm = 0.3 AU/mm. The planet has shifted in position about 4 mm, so this corresponds to 4 x 0.3 = **1.2 AU or 180 million km**.

C – The average speed is 180 million km/17520 hours = **10,000 km/hr.**

D – The radius of the circle is 113 AU so the circumference is $2 \pi R = 2 (3.141) (113 \text{ AU}) = 710$ AU. The distance traveled by the planet in 2 years is, from Problem 2, about 1.2 AU, so in 2 years it traveled 1.2/710 = 0.0017 of its full orbit. That means a full orbit will take 2.0 years/0.0017 = **1,200 years.** Note - Because we are only seeing the 'projected' motion of the planet along the sky, the actual speed could be faster than the estimate in Problem 3, which would make the estimate of the orbit period a bit smaller than what students calculate in Problem 4.

Problem 24 – Working with expanding space

The fact that our universe is expanding was predicted by Einstein's theory of general relativity by the mathematical development of 'Big Bang' cosmology, but was also detected for the first time by astronomer Edwin Hubble. The expansion of the universe means that, although the locations of distant galaxies remain at fixed coordinate positions, the scale of the coordinate grid is increasing. Distances between galaxies increase, not because the galaxies are moving, but because the space between them is stretching or dilating. From any vantage point, this appears as though galaxies are rushing away at speeds the increase as they become more distant. This is called Hubble's Law and represented by the formula V = H d where V is the recession speed in km/s and d is the distance to the galaxy in megaparsecs, where 1 megaparsec = 3,260,000 light years. The current observed value for H is 69 km /s/mpc.

E) An astronomer uses a spectrograph to measure the spectral lines of hydrogen in the galaxy NGC 2342, and finds that they have been shifted by an amount corresponding to a doppler speed of 5,690 km/s. What is the distance to that galaxy in light years? Answer: d = 5690/69 = 82.5 megaparsecs or **270 million light years**.

F) Astronomers use the redshift parameter, z, to indicate the wavelength change caused by the expansion of space according to $z = (\lambda - \lambda r)/\lambda r$ where λ is the observed wavelength in Angstroms and λr is the wavelength of the same spectral line at rest in the laboratory. The recession speed of the object is then V = cz where c = the speed of light 300,000 km/s. An absorption feature of calcium usually has a wavelength of 3934 Å, but it is observed in a galaxy to have a wavelength of 4002 Å. What is the recession velocity? Answer: λ = 4002 λo = 3934 so z = (4002-3934)/4002 = 0.017 so V = 0.017x300000km/s = **5,100 km/s**.

G) The formula in Part B is only accurate if the recession speeds are less than 10% the speed of light. For redshifts of z=1 or greater the relativistic formula below is used. In 1999, astronomers have identified nearly 1000 galaxies with z > 2 and more than 50 galaxies with z > 5. The most remote galaxy at that time was HDF 4-473.0 with a redshift of z=5.6. What is the recession velocity for this infant galaxy? Answer: V = [42.56/44.56]c = 0.955c or **286,000 km/s**.

$$V = \left[\frac{(z+1)^2-1}{(z+1)^2+1}\right] c$$

H) The redshift factor z is also a measure of the amount by which the universe's scale has expanded according to $1+z = R_0/R$ where R is the scale of the universe at the time the light was emitted, and R_0 is the scale of the universe today, represented by R_0=1.0. In 2021 astronomers discovered the farthest known galaxy GN-z11 with a redshift of z=11.4. What was the scale of the universe when its light was first emitted 13.4 billion years ago? Answer: 1+z = 1/R and 1+11.4 = 1/R so **R = 1/12.4**. Objects at that time were 12.4 times closer together than they are today.

Problem 25 – How big is the universe?

Our universe has been expanding for 13.4 billion years, which means that objects that were once close together can now be billions of light years apart. Because the expansion of space is not limited by the speed of light, the distances between objects can change faster than the speed of light as space dilates. The objects are not actually traversing the space between them to reach their large separations. This means that our universe contains regions close enough to us that light could have traversed the distance in the current age of the universe of 13.4 billion years. This region of the universe surrounding us is called the Visible Universe and contains all objects that we now observe. There is a far-vaster universe of galaxies and stars beyond this limit for which we have not as yet received their light since they were formed in the Big Bang. General relativity lets us determine the distances to these objects today, called the comoving distance, from the approximate formula below:

$$d(gLY) = 8.5\ln(z) + 11.7$$

D) The galaxy GN-z11 was recently discovered at a redshift of z=11.4. What is the actual distance to this object today in billions of light years (gLY)? Answer: d = 8.5ln(11.4)+11.7 = **32.4 billion light years.**

E) What is the light travel distance to this galaxy if its light was emitted 13.3 billion years ago? Answer: **13.3 billion light years**.

F) At the present time, what fraction of the current volume of space is occupied by our visible universe based on the distance to GN-z11? Answer: $(13.3/32.4)^3$ = **about 1/14th**.

Problem 26 – Distance and look-back time.

The images we see of distant objects are as they were long ago because of the finite speed of light. For very distant galaxies, this light-travel time is called the 'look-back' time because you are looking back at what an object looked like when light was first emitted by it to form the image you are seeing. Because of the expansion of space as light travels across it, the amount of travel time is related to the redshift of the object being observed. The lookback time is exactly calculated from a cosmological model, but for the observed properties of our universe, it can be approximated by the equation T(gyr) = 2.1ln(z)+8.6. What is the lookback time for the galaxy GN-z11 at a redshift of z=11.4? Answer: **13.7 billion years**. The actual, exact calculation yields 13.3 billion years. For a universe that is 13.7 billion years old, we are seeing the light from this galaxy when the universe was only 400 million years old.

Problem 27 - Seeing the Distant Universe

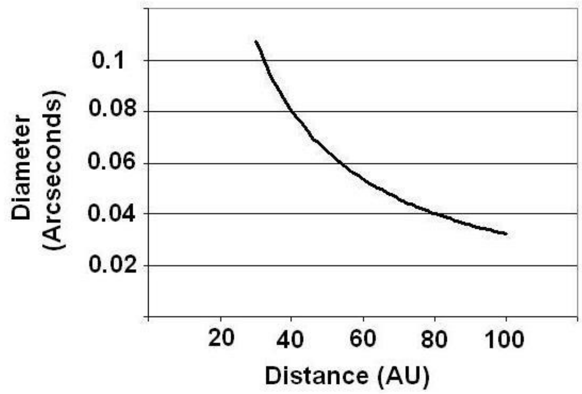
A - $L = \frac{120}{206265} x384,000km$ so L = **220 kilometers**

B - $d = \frac{206265}{0.032} x12,800km$ so **d = 83 billion km**.

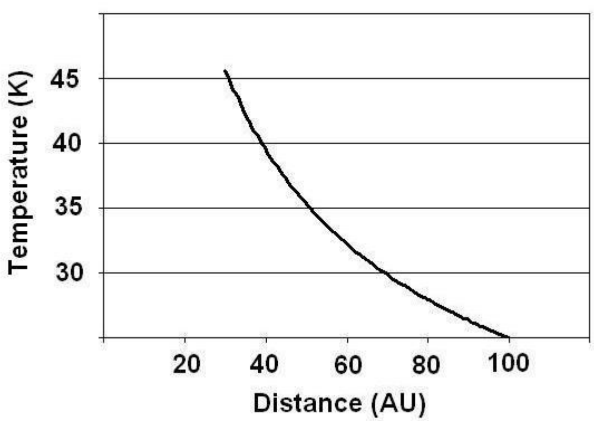
C - $L = \frac{0.032}{206265} x13 \text{ billion light years}$ so **L = 2,000 light years**

Note: no correction has been made for the fact that over these great distances, the curvature of space causes the relationship between angular size and distance to be different than the formula used, which is only valid for the geometry of flat 'Euclidean' space.

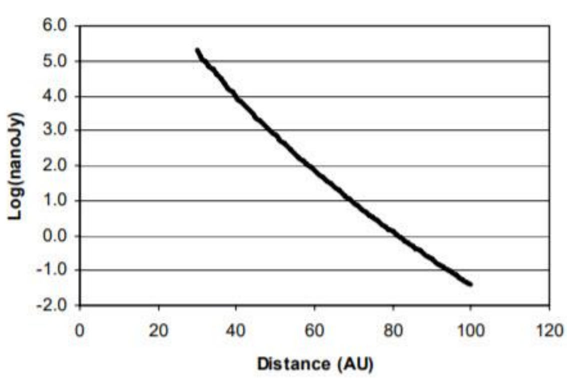
Problem 28 - Webb Space Telescope: Detecting dwarf planets



Problem 1 - At 90 AU, the disk of a Pluto-sized body will be **0.035 arcseconds** in diameter.



B – At 90 AU, the predicted temperature will be about **27 K.**



C - At 4 nanoJanskies, Log(4 nanoJy) = 0.60 which occurs at a distance of about **40 AU.**

Derivation of constant '0.0014' used in Problem 1:

$$\theta = 57.3\frac{L}{R}$$

$$\theta = 206265\frac{L}{R}$$

$$\theta = 206265\frac{L(km)}{149millionR(AU)} = 0.00138\frac{L(km)}{R(AU)}$$

Which is rounded to 2 significant figures as **0.0014**.

Derivation of the constant '720' used in Problem 3

$$B = \frac{2h\upsilon^3}{c^2\left(e^x - 1\right)}$$

$$h = 6.6x10^{-27}\,ergs / Hz$$

$$c = 3x10^{10}\,cm / s$$

$$\upsilon = \frac{3x10^{10}\,cm / s}{20microns} = 1.5x10^{13}\,Hz$$

$$\frac{2h\upsilon^3}{c^2} = 4.95x10^{-8}$$

So this gives B in units of $\frac{ergs}{cm^2 \sec Hz\ Steradians}$

Convert this into SI by multiplying 4.95x10^{-8} x (10^4 cm^2/1 m^2) x (1 joule/10^7 ergs) and where 1 Joule/sec = 1 watt, you get

$$4.95x10^{-11}\frac{Watts}{m^2 HzStr}$$

Astronomers use the unit of 1 Jansky = 10^{-26} Watts/m^2Hz so the final form for B in units of Janskys/steradian is

$$B = \frac{4.95x10^{15}}{(e^x - 1)}\,Jy / str$$

where $x = \frac{h\upsilon}{kT}$ and where ν = 1.5x10^{13}Hz h = Planck's constant = 6.67x10^{-27} ergSec and k= Boltzmann's Constant = 1.38x10^{-16} erg K you get $x = \frac{(6.6x10^{-27})x(1.5x10^{13})}{1.38x10^{-16}T} = \frac{717}{T}$ which to two significant figures is **720/T**

Derivation of constant '120000' used in Problem 1

First, multiply B by the solid angle of the object in steradians θ^2 to get the flux of radiation in Watts/m^2/str and then convert 1 steradian into square arcseconds, and round the answer to 3 significant figures.

$$F = \frac{4.95x10^{15}}{(e^x - 1)}\theta^2 Janskys$$

$$F = \frac{4.95x10^{15}}{206265^2}\frac{\theta^2}{(e^x - 1)} Janskys$$

$$F = \frac{4.95x10^{15}}{206265^2}\frac{\theta^2}{(e^x - 1)} Janskys$$

$$F = 116346\frac{\theta^2}{(e^x - 1)} Janskys$$

$$F = 116000\frac{\theta^2}{(e^x - 1)} Janskys$$

The constant to two significant figures is rounded to **120000.**

Problem 29 - The Cosmological Redshift - Changing the light from a galaxy.

A - λ = 5.0, so 5.0 = 0.5 x (1+z) and so **z = 9.0;** λ = 25, so 25 = 0.5 x (1+z), and so **z = 49.0** The redshift interval is then z = [9,49] The Reionization event can be detected between 3.0 and 4.0 microns, which is within the observing wavelength ranges of both the NIRcam and the FGS-TFC instruments, but not the MIR camera.

B - Answer: z=5.0 corresponds to λ = 0.5 x (1 + 5) so **λ = 3.0 microns**. For z = 7.0 λ = 0.5 x (1 + 7) = **4.0 microns**.

C - The NIRcam is sensitive to radiation between 0.6-5.0 microns, the MIR instrument range is 5.0 to 25.0 microns, and the Fine Guidance Sensor-Tunable Filter Camera detects light between 1 to

5 microns. Which instruments can study the Reionization event in normal galaxies? Answer: **The NIRcam and the Fine Guidance Sensor-Tunable Filter Camera**

IX - Additional resources in infrared science from across NASA

Teaching Space With NASA: Infrared Light

https://scienceandtechnology.jpl.nasa.gov/teaching-space

Infrared Astronomy: More than our eyes can see

https://tinyurl.com/y5gqp2b3

Wide-field Infrared Survey Explorer (WISE)

http://wise.ssl.berkeley.edu/science.html

Your Infrared Guide to the world and the universe beyond

https://coolcosmos.ipac.caltech.edu/

Infrared Waves

https://science.nasa.gov/ems/07_infraredwaves

SpaceMath@NASA Electromagnetic Math...

https://tinyurl.com/2c8t759f

Cool Cosmos: What is Infrared?

https://coolcosmos.ipac.caltech.edu/page/what_is_infrared

Imagine the Universe: The Electromagnetic Spectrum

https://tinyurl.com/zm2ugvg

The Electromagnetic Spectrum Video Series and Companion Book

https://science.nasa.gov/ems

The Electromagnetic Spectrum

https://hubblesite.org/contents/articles/the-electromagnetic-spectrum

Electromagnetic Spectrum: The Musical

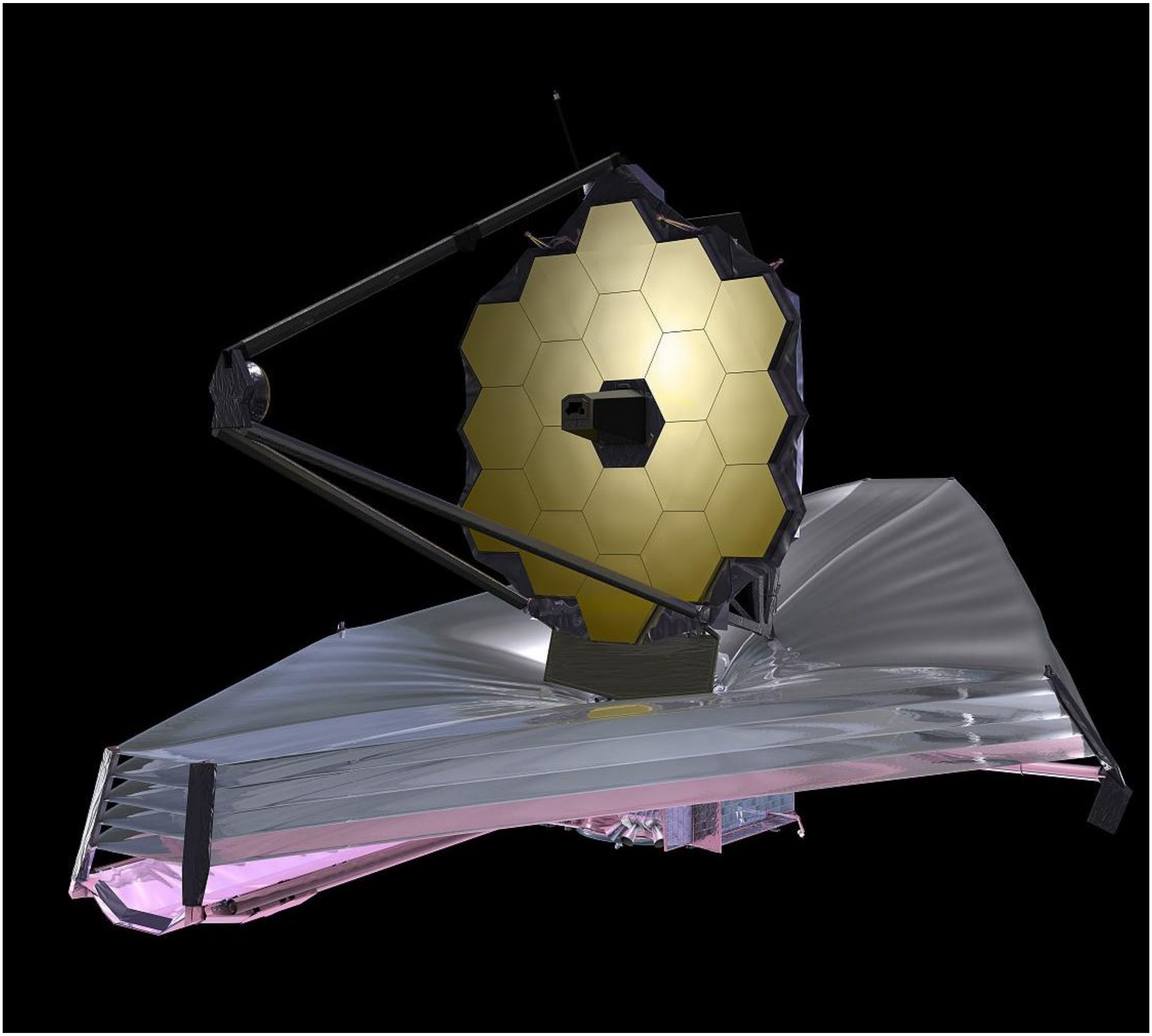
https://www.ipac.caltech.edu/page/117

Multiwavelength Milky Way: Radiation Laws

https://asd.gsfc.nasa.gov/archive/mwmw/mmw_bbody.html

JPL Teach

https://www.jpl.nasa.gov/edu/teach/



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